Complex Numbers

Question1

If a complex number $z = x + iy$ represents a point P on the argand plane and $\arg\left(\frac{z-3+2i}{z+2-3i}\right) = \frac{\pi}{4}$, then the locus of P is a

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Options:

A.

circle with the line $x + y = 12$ as its diameter

B.

circle with radius $\sqrt{11}$

C.

circle with the line $x - y = 6$ as its diameter

D.

circle with radius 5

Answer: D

Solution:

$\because \arg\left(\frac{Z-3+2i}{Z+2-3i}\right) = \frac{\pi}{4}$ let $A = 3 - 2i, B = -2 + 3i$

this means, the angle between $Z - A$ and $Z - B$ is $\pi/4$ i.e., the point Z subtends an angle of $\pi/4$ at the fixed point Z lies on circle that subtends angle $\frac{\pi}{4}$ at the chord joining points A and B.

Now, distance between A and $B = |A - 1|$

$= |(3 - 2i) - (-2 + 3i)| = |5 - 5i|$
$= 5\sqrt{2}$

therefore,

$$\text{Radius}(R) = \frac{AB}{2\sin\theta} = \frac{5\sqrt{2}}{2\sin\pi/4}$$
$$= \frac{5\sqrt{2}}{2 \times \frac{1}{\sqrt{2}}} = 5$$

Hence, the locus of P is a circle with radius 5 .

Question2

By taking $\sqrt{a \pm ib} = x \pm iy, x > 0$, if we get $\frac{\sqrt{21+12\sqrt{2i}}}{\sqrt{21-12\sqrt{2i}}} = a + ib$, then $\frac{b}{a} =$

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Options:

A.

$\frac{4\sqrt{2}}{7}$

B.

$\frac{12\sqrt{2}}{17}$

C.

$\frac{4\sqrt{3}}{7}$

D.

$\frac{12\sqrt{3}}{17}$

Answer: A

Solution:

Let $\sqrt{\frac{A}{B}} = \sqrt{\frac{21+12\sqrt{2}i}{21-12\sqrt{2}i}}$

Now,

$$\frac{21+12\sqrt{2}i}{21-12\sqrt{2}i} = \frac{21+12\sqrt{2}i}{21-12\sqrt{2}i} \times \frac{21+12\sqrt{2}i}{21+12\sqrt{2}i}$$

$$= \frac{(21+12\sqrt{2}i)^2}{(21)^2-(12\sqrt{2}i)^2}$$

$$= \frac{153+504\sqrt{2}i}{729}$$

$$= \frac{153}{729} + \frac{504\sqrt{2}}{729}i$$

then, $\sqrt{\frac{153}{729} + \frac{504\sqrt{2}i}{729}} = a + ib$

Squaring both sides, we get

$$\frac{153}{729} + \frac{504\sqrt{2}}{729}i = a^2 - b^2 + (2ab)i$$

So, $a^2 - b^2 = \frac{153}{729}$ and $ab = \frac{252\sqrt{2}}{729}$

$\Rightarrow a^2 - b^2 = \frac{17}{81}$ and $ab = \frac{28\sqrt{2}}{81}$

let $b = ak$

$\Rightarrow b/a = k$

So, $a^2\left(1-k^2\right) = \frac{17}{81}$ and $a^2 k = \frac{28\sqrt{2}}{81}$

So, $\frac{k}{1-k^2} = \frac{28\sqrt{2}}{81} \times \frac{81}{17} = \frac{28\sqrt{2}}{17}$

From the options, $k = \frac{4\sqrt{2}}{7}$ satisfy the equation.

Question3

Two values of $(-8-8\sqrt{3}i)^{1/4}$ are

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Options:

A.

$\sqrt{3}-i, -1-\sqrt{3i}$

B.

$\sqrt{3}+i, 1+\sqrt{3}i$

C.

$-\sqrt{3}+i, \sqrt{3}+i$

D.

$1-\sqrt{3}i, \sqrt{3}+i$

Answer: A

Solution:

We have, $(-8-8\sqrt{3}i)^{1/4}$

let, $z = -8-8\sqrt{3}i$

$|z| = \sqrt{(-8)^2+(-8\sqrt{3})^2} = 16$

and $\theta = \tan^{-1}\left(\frac{-8\sqrt{3}}{-8}\right) = \pi/3$

so, but the complex number lies in 3rd quadrant

So, $\theta = \pi + \pi/3 = 4\pi/3$

then, $z = 16e^{i4\pi/3}$

So, $z^{1/4} = (16)^{1/4} \cdot e^{i\left(\frac{4\pi}{3}+2\pi k\right)/4}$

$k = 0, 1, 2, 3, \ldots$

for $k = 0$,

$z^{1/4} = 2e^{i\left(\frac{4\pi}{3}\right)/4} = 2e^{i\frac{\pi}{3}} = 1+i\sqrt{3}$

for $k=1$,

$$z^{1/4}=2e^{i\left(\frac{4\pi}{3}+2\pi\right)/4}=2e^{i\left(\frac{\pi}{3}+\frac{\pi}{2}\right)}$$
$$=-\sqrt{3}+i$$

for $k=2$,

$$z^{1/4}=2e^{i\left(\frac{4\pi}{3}+4\pi\right)/4}=2e^{i\left(\pi+\frac{\pi}{3}\right)}$$
$$=-1-i\sqrt{3}$$

for $k=3$,

$$z^{1/4}=2e^{i\left(\frac{4\pi}{3}+6\pi\right)/4}=2e^{i(\pi/3+3\pi/2)}$$
$$=\sqrt{3}-i$$

Hence, two values are $\sqrt{3}-i,-1-\sqrt{3}i$

Question4

If $x=3-2\sqrt{3}\mathrm{i}$, then $x^4-12x^3+54x^2-108x-54=$

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Options:

A.

0

B.

6

C.

-6

D.

9

Answer: D

Solution:

Given, $x=3-2\sqrt{3}i$
$\Rightarrow\quad(x-3)=-2\sqrt{3}i$

Squaring both sides, we get

$$(x-3)^2=(-2\sqrt{3}i)^2$$
$$\Rightarrow x^2-6x+9=-12\Rightarrow x^2-6x=-21$$

Again, squaring both sides, we get

$$\left(x^2-6x\right)^2=441$$

$$\Rightarrow x^4 - 12x^3 + 36x^2 = 441$$

Now, $x^4 - 12x^3 + 54x^2 - 108x - 54$

$$= x^4 - 12x^3 + 36x^2 + 18x^2 - 108x - 54$$
$$= 441 + 18\left(x^2 - 6x - 3\right)$$
$$= 441 + 18(-21 - 3)$$
$$= 441 - 18(24)$$
$$= 441 - 432 = 9$$

Question5

z_1, z_2, z_3 represent the vertices A, B, C of a $\triangle ABC$ respectively in the argand plane. If $|z_1 - z_2| = \sqrt{25 - 12\sqrt{3}}$, $\left|\frac{z_1 - z_3}{z_2 - z_3}\right| = \frac{3}{4}$ and $\angle ACB = 30^\circ$, then the area (in sq units) of that triangle is

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Options:

A.

$\frac{3}{2}$

B.

3

C.

5

D.

$\frac{5}{2}$

Answer: B

Solution:

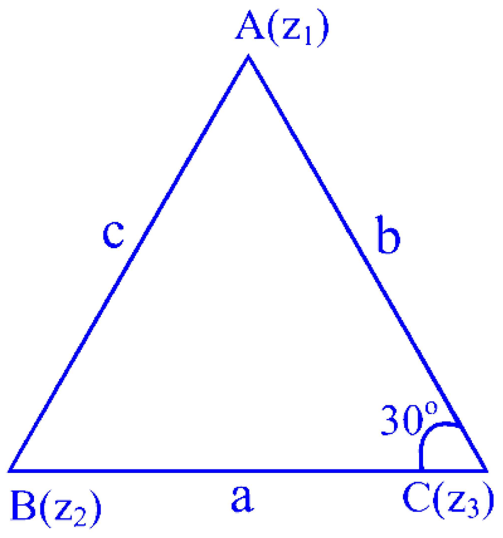
We have,

$$|z_1 - z_2| = \sqrt{25 - 12\sqrt{3}} = c$$

and $\dfrac{|z_1 - z_3|}{|z_2 - z_3|} = \dfrac{3}{4} = \dfrac{b}{a} \angle ACB = 30^\circ$

Let $b = 3k, a = 4k$

$$\because \cos C = \frac{b^2 + a^2 - c^2}{2ab}$$



$$\frac{\sqrt{3}}{2}=\frac{9k^2+16k^2-25+12\sqrt{3}}{24k}$$
$\Rightarrow \quad 12\sqrt{3}k=25k^2-25+12\sqrt{3}$
$\Rightarrow \quad 25k^2-12\sqrt{3}k-25+12\sqrt{3}=0$
$\Rightarrow \quad 25\left(k^2-1\right)-12\sqrt{3}(k-1)=0$
$\Rightarrow \quad (k-1)(25(k+1)-12\sqrt{3})=0$
$\Rightarrow \quad k=1 \text{ or } 25(k+1)=12\sqrt{3}$
$\Rightarrow \quad k=\frac{12\sqrt{3}-25}{25}<0$
$\because k$ cannot be negative.
$\therefore \quad k=1 \Rightarrow b=3, a=4$
$\therefore$ Area of $\triangle ABC=\frac{1}{2}ab\sin C$
$=\frac{1}{2}\times 3\times 4\times \sin 30^{\circ}=3$

Question6

The product of the four values of the complex number $(1+i)^{3/4}$ is

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Options:

A.

$2(1+i)$

B.

$2(1-i)$

C.

$2^3(1+i)$

D.

$2^3(1-i)$

Answer: B

Solution:

Given, $z=(1+i)^{3/4}$

$$=(\sqrt{2})^{\frac{3}{4}}\left(e^{i\frac{\pi}{4}}\right)^{\frac{3}{4}}=2^{\frac{3}{8}}\left(e^{i\frac{3\pi}{4}}\right)^{\frac{1}{4}}$$
$$=2^{3/8}e^{\frac{i}{4}}\left(2n\pi+\frac{3\pi}{4}\right);n=0,1,2,3$$

$\therefore$ Product of four values

$$=\left(2^{3/8}\right)^4 e^{\frac{i}{4}}\left(\frac{3\pi}{4}+2\pi+\frac{3\pi}{4}+4\pi+\frac{3\pi}{4}+6\pi+\frac{3\pi}{4}\right)$$
$$=2^{3/2}\left(e^{i}\frac{15\pi}{4}\right)=2^{3/2}\left(e^{i}4\pi-\frac{\pi}{4}\right)$$
$$=2^{3/2}\left(e^{i4\pi}\right)\cdot e^{i\left(-\frac{\pi}{4}\right)}$$
$$=2^{3/2}(1)\left(\cos\left(\frac{-\pi}{4}\right)+i\sin\left(\frac{-\pi}{4}\right)\right)$$
$$=2\sqrt{2}\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)=2(1-i)$$

Question7

If the point P denotes the complex number $z=x+iy$ in the argand plane and $\frac{z-(2-i)}{z+(1+2i)}$ is purely imaginary number, then the locus of P is

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Options:

A.

a hyperbola not containing the point $(-1,-2)$

B.

an ellipse not containing the point $(-1,-2)$

C.

a parabola not containing the point $(-1,-2)$

D.

a circle not containing the point $(-1,-2)$ and having its centre on the line $x+y+1=0$

Answer: D

Solution:

We note that the quotient $\frac{z-(2-i)}{z+(1+2i)}$ is not defined if $z=-(1+2i) \Leftrightarrow x=-1$ and $y=-2$

Since $x=x+iy$, then

$$\begin{aligned}
&\frac{z-(2-i)}{z+(1+2i)}=\frac{x+iy-(2-i)}{x+iy+(1+2i)} \\
&=\frac{(x-2)+i(y+1)}{(x+1)+i(y+2)} \\
&=\frac{(x-2)+i(y+1)}{(x+1)+i(y+2)} \times \frac{(x+1)-i(y+2)}{(x+1)-i(y+2)} \\
&=\frac{(x-2)(x+1)-i(x-2)(y+2)}{+i(y+1)(x+1)+(y+1)(y+2)} \\
&=\frac{[(x-2)(x+1)+(y+1)(y+2)]}{(x+1)^2+(y+2)^2}+\frac{i[(x+1)(y+1)-(x-2)(y+2)]}{(x+1)^2+(y+2)^2}
\end{aligned}$$

Since, $\frac{z-(2-i)}{z+(1+2i)}$ is purely imaginary,

So real part $=0$

$$\begin{aligned}
&\therefore \frac{(x-2)(x+1)+(y+1)(y+2)}{(x+1)^2+(y+2)}=0 \\
&\Leftrightarrow x^2+x-2x-2+y^2+2y+y+2=0 \\
&x^2+y^2-x+3y=0
\end{aligned}$$

And $x \neq -1$ and $y \neq -2$

This is the locus of point P. This is a circle.

Question8

If $(\sqrt{3}-i)^n=2^n, n \in N$, then the least possible value of n is

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Options:

A.

3

B.

4

C.

6

D.

12

Answer: D

Solution:

Given, $(\sqrt{3}-i)^n, n \in N$

Modulus $|\sqrt{3}-1| = \sqrt{(\sqrt{3})^2+(-1)^2}$

$$= \sqrt{3+1} = \sqrt{4} = 2$$

$\theta = \arg(\sqrt{3}-i) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}$

Argument :

$\theta = \arg(\sqrt{3}-i) = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{-\pi}{6}$

So, in polar form,

$$\begin{aligned}\sqrt{3}-i &= 2\cdot(\cos\theta + i\sin\theta)\\ &= 2\left[\cos\left(\frac{-\pi}{6}\right)+\sin\left(\frac{-\pi}{6}\right)\right]\\ &= 2\left[\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right]\end{aligned}$$

$$\Rightarrow (\sqrt{3}-i)^n = \left\{2\cdot\left[\cos\left(\frac{\pi}{6}\right) - i\sin\left(\frac{\pi}{6}\right)\right]\right\}^n$$

$$= 2^n\cdot\left[\cos\left(\frac{n\pi}{6}\right) - i\sin\left(\frac{n\pi}{6}\right)\right] \qquad \ldots(i)$$

But given that $(\sqrt{3}-1)^n = 2^n \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$\begin{aligned}&2^n\left[\cos\left(\tfrac{n\pi}{6}\right) - i\sin\left(\tfrac{n\pi}{6}\right)\right] = 2^n\\ \Rightarrow\ &\cos\left(\tfrac{n\pi}{6}\right) - i\sin\left(\tfrac{n\pi}{6}\right) = 1\\ \Rightarrow\ &\cos\left(\tfrac{n\pi}{6}\right) = 1 \text{ and } \sin\left(-\tfrac{n\pi}{6}\right) = 1\\ \Rightarrow\ &-\tfrac{n\pi}{6} = 2k\pi\\ \Rightarrow\ &n = \tfrac{-12k\pi}{\pi} = -12k\end{aligned}$$

Since, $n \in N$, for the smallest positive n, let $k=-1$

$\Rightarrow \quad n = -12(-1) = 12$

$\therefore$ The least positive value of n is 12 .

Question9

$$(1+\sqrt{5}+i\sqrt{10-2\sqrt{5}})^5 =$$

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Options:

A.

1024

B.

-1024

C.

512

D.

-512

Answer: B

Solution:

Let $z=1+\sqrt{5}+i\sqrt{10-2\sqrt{5}}$

$$
\begin{aligned}
So, |z| &= \sqrt{(1+\sqrt{5})^2+(\sqrt{10-2\sqrt{5}})^2} \\
&= \sqrt{1+5+2\sqrt{5}+10-2\sqrt{5}} \\
&= \sqrt{16+2\sqrt{5}-2\sqrt{5}}=\sqrt{16}=4 \\
\text{And } \theta = \arg(z) &= \tan^{-1}\left(\frac{\sqrt{10-2\sqrt{5}}}{1+\sqrt{5}}\right) \\
&= \tan^{-1}\left(\tan\left(\frac{\pi}{5}\right)\right)=\frac{\pi}{5}
\end{aligned}
$$

So, $z=|z|\cdot(\cos\theta+i\sin\theta)$

$$
\begin{aligned}
&= 4\left[\cos\left(\frac{\pi}{5}\right)+i\sin\left(\frac{\pi}{5}\right)\right] \\
\therefore z^5 &= 4^5\left[\cos\left(\frac{\pi}{5}\right)+i\sin\left(\frac{\pi}{5}\right)\right]^5 \\
&= 1024\left[\cos\left(5\cdot\frac{\pi}{5}\right)+i\sin\left(5\cdot\frac{\pi}{5}\right)\right] \\
&= 1024[\cos\pi+i\sin\pi] \\
&= 1024[-1+0]=-1024
\end{aligned}
$$

Question10

If z is a complex number such that $\frac{z-1}{z-i}$ is purely imaginary and locus of z represents a circle with centre (α,β) and radius r, then $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=$

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Options:

A.

$4r$

B.

r^2

C.

$2r^2$

D.

$4r^2$

Answer: D

Solution:

$$\begin{aligned}\frac{z-1}{z-i} &= \frac{(x+iy)-1}{(x+iy)-i} \\ &= \frac{(x-1)+iy}{x+i(y-1)} \\ &= \frac{(x-1)+iy}{x+i(y-1)} \times \frac{(x-i(y-1))}{(x-i(y-1))}\end{aligned}$$

$= \frac{[x(x-1)+y(y-1)]+i[xy-(x-1)(y-1)]}{x^2+(y-1)^2}$

$\because$ The fraction is purely imaginary.

So, $x^2 - x + y^2 - y = 0$ {real part should be zero)

$$\begin{aligned}&\Rightarrow \quad x^2 - x + \frac{1}{4} + y^2 - y + \frac{1}{4} = \frac{1}{4} + \frac{1}{4} \\ &\Rightarrow \quad \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2\end{aligned}$$

Center of circle $(\alpha, \beta) = \left(\frac{1}{2}, \frac{1}{2}\right)$

and Radius $(r) = \frac{1}{\sqrt{2}}$

Thus, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(1/2)}{(1/2)} + \frac{(1/2)}{(1/2)} = 2$

and $r = \frac{1}{\sqrt{2}}$

$\Rightarrow r^2 = \frac{1}{2}$

So, $4r^2 = 4 \times \frac{1}{2} = 2$

Hence, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 4r^2$

Question11

If the least positive integer n satisfying the equation $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^n = -1$ is p and the least positive integer m satisfying the equation $\left(\frac{1-\sqrt{3i}}{1+\sqrt{3i}}\right)^m = \text{cis}\,\frac{2\pi}{3}$ is q, then $\sqrt{p^2+q^2} =$

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Options:

A.

5

B.

10

C.

$\sqrt{13}$

D.

$\sqrt{17}$

Answer: C

Solution:

$\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^n = -1$

$\because$ Modulus of numerator $= \sqrt{(\sqrt{3})^2 + 1^2}$

$= 2\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

and modulus of denominator $= 2$

$$\theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$
$$\begin{aligned}\therefore \left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^n &= \left(\frac{2\operatorname{cis}\left(\frac{\pi}{6}\right)}{2\operatorname{cis}\left(-\frac{\pi}{6}\right)}\right)^n \\ &= \left[\operatorname{cis}\left(\frac{\pi}{6} - \left(-\frac{\pi}{6}\right)\right)\right]^n \\ &= \left[\operatorname{cis}\left(\frac{\pi}{3}\right)\right]^n = \operatorname{cis}\left(\frac{n\pi}{3}\right)\end{aligned}$$

$$\begin{aligned}&\text{and } \operatorname{cis}\left(\frac{n\pi}{3}\right) = -1 \\ &= \operatorname{cis}(\pi) \\ \therefore \quad & \frac{n\pi}{3} = \pi \\ \Rightarrow \quad & n = 3\end{aligned}$$

thus, $p = 3$

Similarly, second expression

$$\left(\frac{1-\sqrt{3}i}{1+\sqrt{3}i}\right)^m = \operatorname{cis}\left(\frac{2\pi}{3}\right)$$
$$\Rightarrow \left[\frac{2\left(\operatorname{cis}\left(-\frac{\pi}{3}\right)\right)}{2\left(\operatorname{cis}\left(\frac{\pi}{3}\right)\right)}\right]^m = \operatorname{cis}\left(\frac{2\pi}{3}\right)$$
$$\Rightarrow \left[\operatorname{cis}\left(-\frac{\pi}{3} - \frac{\pi}{3}\right)\right]^m = \operatorname{cis}\left(\frac{2\pi}{3}\right)$$
$$\Rightarrow \operatorname{cis}\left(-\frac{2\pi}{3}\cdot m\right) = \operatorname{cis}\left(\frac{2\pi}{3}\right)$$
$$\text{Therefore, } -\frac{2\pi}{3}\cdot m = \frac{2\pi}{3}$$

$$\begin{aligned}\Rightarrow \quad & -2\pi m \equiv 2\pi \cdot \bmod (6\pi) \\ \Rightarrow \quad & -2m \equiv 2 \bmod (6) \\ \Rightarrow \quad & 2m \equiv -2 \equiv 4 \bmod 6 \\ \Rightarrow \quad & m = 2 \bmod 3\end{aligned}$$

$\therefore$ Least positive integer $= m = 2$ thus, $q = 2$

Hence, $\sqrt{q^2 + p^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$

Question12

Sum of the squares of the imaginary roots of the equation $z^8 - 20z^4 + 64 = 0$ is

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Options:

A.

0

B.

-12

C.

-4

D.

-16

Answer: B

Solution:

$$\begin{aligned} & z^8 - 20z^4 + 64 = 0 \\ \Rightarrow \quad & \left(z^4\right)^2 - 16z^4 - 4z^4 + 64 = 0 \\ \Rightarrow \quad & \left(z^4 - 16\right)\left(z^4 - 4\right) = 0 \\ & z^4 = 16 \text{ or } z^4 = 4 \\ \Rightarrow \quad & z = \pm 2 \pm 2i, \pm\sqrt{2}, \pm\sqrt{2}i \end{aligned}$$

imaginary roots are $2i, -2i, \sqrt{2}i, -\sqrt{2}i$ sum of square the imaginary roots

$$\begin{aligned} & = (2i)^2 + (-2i)^2 + (\sqrt{2}i)^2 + (-\sqrt{2}i)^2 \\ & = -4 - 4 - 2 - 2 = -12 \end{aligned}$$

Question13

For any two non-zero complex numbers z_1 and z_2, if $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$, then

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Options:

A.

$\operatorname{Re}\left(\frac{z_1}{z_2}\right) = 0$

B.

$\operatorname{lm}\left(\frac{z_1}{z_2}\right)=0$

C.

$\operatorname{Re}\left(z_1 z_2\right)=0$

D.

$\operatorname{lm}\left(z_1 z_2\right)=0$

Answer: A

Solution:

We are given that:

$$|z_1+z_2|^2=|z_1|^2+|z_2|^2$$

For two non-zero complex numbers z_1 and z_2, we know that:

$$|z_1+z_2|^2=(z_1+z_2)(\overline{z_1+z_2})=|z_1|^2+2\operatorname{Re}(z_1\overline{z_2})+|z_2|^2$$

Given that:

$$|z_1+z_2|^2=|z_1|^2+|z_2|^2$$

we can equate:

$$|z_1|^2+2\operatorname{Re}(z_1\overline{z_2})+|z_2|^2=|z_1|^2+|z_2|^2$$

Simplifying:

$$2\operatorname{Re}(z_1\overline{z_2})=0$$

Thus, the **real part** of $z_1\overline{z_2}$ must be zero:

$$\operatorname{Re}(z_1\overline{z_2})=0$$

This means that $z_1\overline{z_2}$ is purely imaginary, implying that the real part of $\frac{z_1}{z_2}$ is also zero.

Final Answer:

The correct answer is **Option A**, which states:

$$\operatorname{Re}\left(\frac{z_1}{z_2}\right)=0$$

Question14

If $1, \omega, \omega^2$ are the cube roots of unity, then

$1\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+2\left(3+\frac{1}{\omega}\right)\left(3+\frac{1}{\omega^2}\right)+3\left(4+\frac{1}{\omega}\right)\left(4+\frac{1}{\omega^2}\right)+\ldots 10 \text{ terms} \quad =$

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Options:

A.

3080

B.

3465

C.

3175

D.

3715

Answer: B

Solution:

Given, $1, \omega, \omega^2$ are cube roots of unity.

Also, we can write the general term (t_n) of given expression

$$\begin{aligned} t_n &= (n-1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right) \\ &= (n-1)\left(n+\omega^2\right)(n+\omega) \\ &= (n-1)\left(n^2+n\left(\omega+\omega^2\right)+\omega^3\right) \\ &= (n-1)\left(n^2-n+1\right)\left[\because 1+\omega+\omega^2=0\right] \\ &= \left(n^3-2n^2+2n-1\right) \qquad \ldots(i) \end{aligned}$$

Now, $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$,

$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{r=1}^{n} r^3 = \left(\frac{n(n+1)}{2}\right)^2$$

So, we have;

$$\begin{aligned} \sum_{n=2}^{11} n &= \left(\sum_{n=2}^{11} n - 1\right) \\ &= \frac{11(12)}{2} - 1 = 66 - 1 = 65 \end{aligned}$$

$$\sum_{n=2}^{11} n^2 - \left(\sum_{n=1}^{11} n^2\right) - 1$$

$$= \frac{11(12)(23)}{6} - 1 = 506 - 1 = 505$$

$$\begin{aligned} \sum_{n=2}^{11} n^3 &= \left(\sum_{n=1}^{11} n^3\right) - 1 \\ &= \left(\frac{11(12)}{2}\right)^2 - 1 = (66)^2 - 1 \\ &= 4356 - = 4355 \end{aligned}$$

$$and \ \sum_{n=2}^{11} 1 = 10$$

So, final answer

$= 4355 - 2(505) + 2(65) - 10$
$= 4355 - 1010 + 130 - 10 = 3465$

Question15

$(1 + \sqrt{3}i)^6 - (\sqrt{3} + i)^6 =$

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Options:

A.

0

B.

32

C.

64

D.

128

Answer: D

Solution:

$$(1 + \sqrt{3}i)^6 - (\sqrt{3} + i)^6$$
$$= 2^6 \cdot \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^6 - \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^6 2^6$$
$$= 2^6\left[\left(\cos\frac{\pi}{3} + \sin\frac{\pi}{3}i\right)^6 - \left(\cos\frac{\pi}{6} + \sin\frac{\pi}{6}i\right)\right]$$

(Using De-Moivre's theorem)

$$= 2^6[(\cos 2\pi + \sin 2\pi i) - (\cos \pi + \sin \pi i)]$$
$$= 64[(1 + 0) - (-1 + 0)]$$
$$= 64(1 + 1) = 128$$

Question16

If $z = x + iy$ and $x^2 + y^2 = 1$, then $\frac{1+x+iy}{1+x-iy} =$

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Options:

A.

$\bar{z}$

B.

z

C.

$z+1$

D.

$z-1$

Answer: B

Solution:

$$
\begin{aligned}
&z=x+iy \text{ and } x^2+y^2=1\\
&\frac{1+x+iy}{1+x-iy}=\frac{((1+x)+iy)((1+x)+iy)}{(1+x)^2-i^2y^2}\\
&\qquad=\frac{(1+x)^2+i^2y^2+2(1+x)yi}{1+x^2+2x+y^2}\\
&\qquad=\frac{1+x^2+2x-y^2+2(1+x)yi}{2+2x}\\
&\qquad=\frac{2x^2+2x+2(1+x)yi}{2(1+x)}\\
&\qquad=\frac{2x(1+x)+2(1+x)yi}{2(1+x)}\\
&\qquad=x+iy=z
\end{aligned}
$$

Question17

If $x^6=(\sqrt{3}-i)^5$, then the product of all of its roots is

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Options:

A.

$2^5(\sqrt{3}+i)$

B.

$\frac{2^6}{\sqrt{3}+i}$

C.

$2^6(\sqrt{3}-i)$

D.

$\frac{2^6}{\sqrt{3}-i}$

Answer: D

Solution:

$$\sqrt{3}-i=2\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$$
$$=2e^{-i\pi/6}$$
$$\Rightarrow(\sqrt{3}-i)^5=\left(2e^{-\frac{i\pi}{6}}\right)^5=32e^{-\frac{i5\pi}{6}}$$

The product of the root of $x^6=c$ is

$(-1)^{6-1}c=-c$

The product of the root is $-32e^{-\frac{i5\pi}{6}}$

$$=-32\left(\cos\left(\frac{-5\pi}{6}\right)+i\sin\left(-\frac{5\pi}{6}\right)\right)$$
$$=-32\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)$$
$$=16\sqrt{3}+16i$$
$$=\frac{2^6}{\sqrt{3}-i}$$

Question18

The minimum value of $|z-1|+|z-5|$ is

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Options:

A.

3

B.

5

C.

4

D.

2

Answer: C

Solution:

$|z-1|+|z-5| \geq |z-1-z+5|$
$\geq 4 [\because |z_1|+|z_2| \geq |z_1-z_2|]$
Minimum value $=4$

Question19

If $z=x+iy$ and if the point P in the argand diagram represents z, then the locus of the point P satisfying the equation $2|z-2-3i|=3|z+i-2|$ is a circle with centre

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Options:

A.

$(10,-21)$

B.

$(2,-\frac{21}{5})$

C.

$(-10,21)$

D.

$(-2,\frac{21}{5})$

Answer: B

Solution:

$z=x+iy$

$2|z-2-3i|=3|z+i-2|$

Let $z=x+iy$

$2|(x-2)+i(y-3)|$

$=3|(x-2)+i(y+1)|$

Squaring on both sides, we get

$4x^2-16x+16+4y^2-24y+36$

$=9x^2-36x+36+9y^2+18y+9$

$\Rightarrow \quad 5x^2+5y^2-20x+42y-7=0$

Centre $\equiv \left(2, \frac{-21}{5}\right)$

Question20

If z is a non-real root of $x^7=1$, then $1+3z+5z^2+7z^3+9z^4+11z^5+13z^6=$

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Options:

A.

$\frac{14}{1-z}$

B.

$\frac{-14}{1-z}$

C.

$\frac{15}{1-z}$

D.

$\frac{-15}{1-z}$

Answer: B

Solution:

$$x^7 = 1$$
$$\Rightarrow x^7 - 1 = 0$$
$$\therefore z^7 - 1 = 0$$
$$(z-1)\left(1+z+z^2+z^3+z^4+z^5+z^6\right) = 0$$
$$z \neq 1$$
$$1+z+z^2+z^3+z^4+z^5+z^6 = 0 \qquad \ldots(i)$$

Now,

$$S = 1+3z+5z^2+7z^3+9z^4+11z^5+13z^6$$
$$zS = z+3z^2+5z^3+7z^4+9z^5+11z^6+13z^7$$
$$- \quad - \quad - \quad - \quad - \quad - \quad - \quad -$$
$$S(1-z) = 1+2z+2z^2+\ldots+2z^6-13z^7$$
$$S(1-z) = -14 \Rightarrow S = \frac{-14}{1-z} \qquad [z^7 = 1]$$

Question21

If $\cosh 2x = 199$**, then** $\cot hx =$

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Options:

A.

$\frac{5}{3\sqrt{11}}$

B.

$\frac{5}{6\sqrt{11}}$

C.

$\frac{7}{3\sqrt{11}}$

D.

$\frac{10}{3\sqrt{11}}$

Answer: D

Solution:

$$\begin{aligned} &\text{If } \cosh 2x = 199 \\ &\qquad \coth x = \sqrt{\frac{1+\cosh 2x}{\cosh 2x - 1}} = \sqrt{\frac{200}{198}} \\ &\Rightarrow \quad \coth 2x = \frac{10}{3\sqrt{11}} \end{aligned}$$

Question22

If $a = \operatorname{Im}\left(\frac{1+z^2}{2iz}\right)$ and z is any non-zero complex number such that $|z| = 1$, then $a =$

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Options:

A.

$\operatorname{Re}(z)$

B.

$\operatorname{Re}(z) \operatorname{Im}(z)$

C.

$-\operatorname{Re}(z)$

D.

$\operatorname{Re}(z) + \operatorname{Im}(z)$

Answer: C

Solution:

We have, $|z| = 1$

$\because$ Let $z = \cos\theta + i\sin\theta = e^{i\theta}$

$$\begin{aligned} 1 + z^2 &= 1 + \cos 2\theta + i \sin 2\theta \\ &= 2\cos^2\theta + 2i \sin\theta \cos\theta \\ &= 2\cos\theta(\cos\theta + i\sin\theta) = 2\cos\theta \cdot e^{i\theta} \end{aligned}$$

Now, $\dfrac{1+z^2}{2iz} = \dfrac{2\cos\theta \cdot e^{i\theta}}{2ie^{i\theta}} = \dfrac{\cos\theta}{i} = -i\cos\theta$

Now, $\operatorname{Im}\left(\dfrac{1+z^2}{2iz}\right) = -\cos\theta = -\operatorname{Re}(z)$

Question23

If $(3+4i)^{2025} = 5^{2023}(x+iy)$, then $\sqrt{x^2+y^2} =$

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Options:

A.

5

B.

25

C.

125

D.

625

Answer: B

Solution:

Given, $(3+4i)^{2025} = 5^{2023}(x+iy)$

Taking modulus on both sides, we get

$$5^{2025} = 5^{2023}\sqrt{x^2+y^2}$$

$\because \sqrt{x^2+y^2} = 5^2 = 25$

Question24

If $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^{2024} + \left(\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta+i\sin\theta}\right)^{2025} = x + iy$ then the value of $x + y$ at $\theta = \frac{\pi}{2}$ is

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Options:

A.

1

B.

-1

C.

2

D.

2024

Answer: C

Solution:

We have, $\left(\frac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta}\right)^{2024}+\left(\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta+i\sin\theta}\right)^{2025}$

$=\left\{\frac{(\cos\theta+i\sin\theta)}{i(\cos\theta-i\sin\theta)}\right\}^{2024}+\left(\frac{2\cos^2\frac{\theta}{2}+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)^{2025}$

$=\left(\frac{e^{i\theta}}{e^{-i\theta}}\right)^{2024}+\frac{\left(2\cos\frac{\theta}{2}\right)^{2025}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^{2025}}{\left(2\sin\frac{\theta}{2}\right)^{2025}\left(\sin\frac{\theta}{2}+i\cos\frac{\theta}{2}\right)^{2025}}$

$=\left(e^{2i\theta}\right)^{2024}+\left(\cot\frac{\theta}{2}\right)^{2025}\frac{\left(e^{i\frac{\theta}{2}}\right)^{2025}}{i^{2025}\left(e^{-i\frac{\theta}{2}}\right)^{2025}}$

$=\left(e^{2i\theta}\right)^{2024}+\left(\cot\frac{\theta}{2}\right)^{2025}\frac{1}{i}\left(e^{i\theta}\right)^{2025}$

$=(\cos 2\theta+i\sin 2\theta)^{2024}-\left(\cot\frac{\theta}{2}\right)^{2025}i(\cos\theta+i\sin\theta)^{2025}$

Now, put $\theta=\frac{\pi}{2}$

$=(-1)^{2024}-i(i)^{2025}=1-i^2=1+1=2$

Question25

If $a\pm ib$ and $b\pm ai$ are the roots of $x^4-10x^3+50x^2-130x+169=0$, then $\frac{a}{b}+\frac{b}{a}=$

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Options:

A.

$\frac{25}{12}$

B.

$\frac{5}{2}$

C.

$\frac{13}{6}$

D.

$\frac{34}{15}$

Answer: C

Solution:

Given,

$x^4-10x^3+50x^2-130x+169=0$

Sum of roots $2a+2b=10 \Rightarrow a+b=5 \quad \ldots(i)$

Product of roots $\left(a^2+b^2\right)\left(b^2+a^2\right)=169$

$a^2+b^2=13 \quad \ldots(ii)$

From, Eq.s (i) and (ii) $ab=6$

$\frac{a^2+b^2}{ab}=\frac{13}{6} \Rightarrow \frac{a}{b}+\frac{b}{a}=\frac{13}{6}$

Question26

If $i=\sqrt{-1}$, then $\sum_{n=2}^{30} i^n+\sum_{n=30}^{65} i^{n+3}=$

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Options:

A.

0

B.

-1

C.

i

D.

-1

Answer: B

Solution:

We have, $i=\sqrt{-1}$

$$\sum_{n=2}^{30} i^n = i^2+i^3+\ldots+i^{30} = 0+i^{30} = 0-1$$

$$\sum_{n=30}^{65} i^{n+3} = i^{33}+i^{34}+\ldots+i^{68} = 0$$

$$\because \sum_{n=2}^{30} i^n + \sum_{n=30}^{65} i^{n+3} = -1+0=-1$$

Question27

If z_1 and z_2 are two of the nth roots of unity such that the line segment joining them subtends at a right angle at the origin, then for a positive integer k, n takes the form

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Options:

A.

$4k$

B.

$4k+1$

C.

$4k+2$

D.

$4k+3$

Answer: A

Solution:

z_1 and z_2 are both nth roots of unity. This means each one can be written in the form $e^{\frac{i2\pi r}{n}}$, where r is an integer.

Let $z_1 = e^{\frac{i2\pi r_1}{n}}$ and $z_2 = e^{\frac{i2\pi r_2}{n}}$ for some integers r_1 and r_2.

If the line segment joining z_1 and z_2 makes a right angle at the origin, this means the points z_1, z_2, and the origin form a right-angled triangle with the right angle at the origin.

This can only happen if $\frac{z_1}{z_2}$ is a purely imaginary number. A complex number is purely imaginary when its argument (angle) is $\frac{\pi}{2}$ or $\frac{3\pi}{2}$, or in general, $\frac{\pi}{2}+m\pi$ where m is an integer.

We find: $\frac{z_1}{z_2} = e^{\frac{i2\pi}{n}(r_1-r_2)}$

For this to be purely imaginary, we need: $\frac{2\pi}{n}(r_1-r_2) = \frac{\pi}{2}+m\pi$

Divide both sides by π: $\frac{2}{n}(r_1 - r_2) = \frac{2m+1}{2}$

Now solve for n: $n = \frac{4(r_1 - r_2)}{2m+1}$

Since n must be a positive integer, $2m+1$ must divide evenly into $4(r_1 - r_2)$. The simplest way for this to always work for integer values is when n is a multiple of 4.Therefore, $n = 4k$ for some positive integer k.

Question28

$$(\sqrt{\sqrt{2}+1} + i\sqrt{\sqrt{2}-1})^8 =$$

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Options:

A.

64

B.

$64i$

C.

-64

D.

$-64i$

Answer: C

Solution:

Let, $Z = \sqrt{\sqrt{2}+1} + i\sqrt{\sqrt{2}-1}$

$$\Rightarrow \quad |Z| = \sqrt{\sqrt{2}+1+\sqrt{2}-1} = \sqrt{2\sqrt{2}}$$

$$\cos\theta = \frac{\sqrt{\sqrt{2}+1}}{\sqrt{2\sqrt{2}}}, \sin\theta = \frac{\sqrt{\sqrt{2}-1}}{\sqrt{2\sqrt{2}}}$$

$$\therefore \quad \cos^2\theta = \frac{\sqrt{2}+1}{2\sqrt{2}}, \sin^2\theta = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

$$\Rightarrow \frac{1+\cos 2\theta}{2} = \frac{\sqrt{2}+1}{2\sqrt{2}}$$

$$\Rightarrow \cos 2\theta = \frac{1}{\sqrt{2}}$$

$\Rightarrow 2\theta = \frac{\pi}{4}$ (since, real and imaginary part are positive.)

Now,

$Z^8 = (\sqrt{2\sqrt{2}})^8 \left(\cos\left(8 \times \frac{\pi}{8}\right) + i\sin\left(8 \times \frac{\pi}{8}\right)\right)$

$= (2\sqrt{2})^4(\cos\pi + i\sin\pi)$
$= \left(2^{\frac{3}{2}} \times 4\right)(-1+0)$
$= 2^6(-1) = -64$

Question29

ω is a complex cube root of unity and if z is a complex number satisfying $|z-1| \leq 2$ and $\left|\omega^2 z - 1 - \omega\right| = a$, then the set of possible values of a is

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Options:

A. $0 \leq a \leq 2$

B. $\frac{1}{2} \leq a \leq \frac{\sqrt{3}}{2}$

C. $|\omega| \leq a \leq \frac{\sqrt{3}}{2} + 2$

D. $0 \leq a \leq 4$

Answer: D

Solution:

We have, $|z-1| \leq 2 \quad \ldots(i)$

and $\left|\omega^2 z - 1 - \omega\right| = a$

$\omega^2 z + \omega^2 \mid = a \quad [\because \omega^2 = -1 - \omega]$

$\left|\omega^2\right||z+1| = a$

$|z-1+2| = a$

$|z-1| + 2 \geq a$

From Eq. (i), we get

$|z-1| + 2 = 4$

$\Rightarrow \quad 0 \leq a \leq 4$

Question30

If the roots of the equation $z^3 + iz^2 + 2i = 0$ are the vertices of a $\triangle ABC$, then that $\triangle ABC$ is

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Options:

A. a right angled triangle

B. an equilateral triangle

C. an isosceles triangle

D. a right angled isosceles triangle

Answer: C

Solution:

We have, $z^3+iz^2+2i=0$

One root of $z^3+iz^2+2i=0$ is i.

$$
\begin{aligned}
& \left[\because i^3+i\left(i^2\right)+2i=-i-i+2i=0\right] \\
& z^3+iz^2+2i=0 \\
\Rightarrow \quad & (z-i)\left(z^2+2iz-2\right)=0 \\
\Rightarrow \quad & (z-i)\left(z^2+2iz+i^2+1-2\right)=0 \\
\Rightarrow \quad & (z-i)\left((z+i)^2-1\right)=0 \\
\Rightarrow \quad & (z-i)(z+(i+1))(z+(i-1))=0 \\
\Rightarrow \quad & z=i,-i-1,1-i
\end{aligned}
$$

$\therefore$ The triangle formed by the vertices $A(0,1), B(-1,-1)$ and $C(1,-1)$.

$AB=\sqrt{1+4}=\sqrt{5}$ units

$BC=2$ units

$AC=\sqrt{1+4}=\sqrt{5}$ units

$\Rightarrow \triangle ABC$ is an isosceles triangle.

$[\because AB=AC]$

Since, the non-equal side is less than the equal side, so ABC is not a right-angled triangle.

Question31

(r,θ) denotes $r(\cos\theta+i\sin\theta)$. If $x=(1,\alpha), y=(1,\beta), z=(1,\gamma)$ and $x+y+z=0$, then $\Sigma\cos(2\alpha-\beta-\gamma)$ is equal to

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Options:

A. 3

B. 0

C. 1

D. -1

Answer: A

Solution:

In the complex plane, any point can be represented in polar form as $(r,\theta)=r(\cos\theta+i\sin\theta)$. Here, $x=(1,\alpha)$, $y=(1,\beta)$, and $z=(1,\gamma)$. Given the condition $x+y+z=0$, we can express

$x = \cos\alpha + i\sin\alpha$

Similarly,

$y = \cos\beta + i\sin\beta$

$z = \cos\gamma + i\sin\gamma$

These imply:

$x + y + z = 0 \Rightarrow \cos\alpha + \cos\beta + \cos\gamma = 0$

$\sin\alpha + \sin\beta + \sin\gamma = 0$

From the above, we isolate:

$\cos\alpha + \cos\beta = -\cos\gamma \quad \text{...(i)}$

$\sin\alpha + \sin\beta = -\sin\gamma \quad \text{...(ii)}$

Squaring and adding equations (i) and (ii), we get:

$$(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2 = \cos^2\gamma + \sin^2\gamma$$

$\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta = 1$

This simplifies to:

$2\cos(\alpha - \beta) = -1 \quad \Rightarrow \quad \cos(\alpha - \beta) = -\frac{1}{2}$

By symmetry,

$\cos(\beta - \gamma) = -\frac{1}{2}$

and

$\cos(\gamma - \alpha) = -\frac{1}{2}$

The corresponding sine values are:

$\sin(\alpha - \beta) = \sin(\beta - \gamma) = \sin(\gamma - \alpha) = \frac{\sqrt{3}}{2}$

Consider $\cos(2\alpha - \beta - \gamma)$:

$$\begin{aligned}\cos(2\alpha - \beta - \gamma) &= \cos((\alpha - \beta) - (\gamma - \alpha)) \\ &= \cos(\alpha - \beta)\cos(\gamma - \alpha) + \sin(\alpha - \beta)\sin(\gamma - \alpha) \\ &= \left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 \\ &= \frac{1}{4} + \frac{3}{4} \\ &= 1\end{aligned}$$

Similarly,

$\cos(2\beta - \gamma - \alpha) = 1$

$\cos(2\gamma - \alpha - \beta) = 1$

Thus, we find:

$\Sigma\cos(2\alpha - \beta - \gamma) = 1 + 1 + 1 = 3$

Question32

$\arg\left[\frac{(1+i\sqrt{3})(-\sqrt{3}-i)}{(1-i)(-i)}\right]$ is equal to

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Options:

A. $\frac{5\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{2\pi}{3}$

D. $\frac{-\pi}{2}$

Answer: B

Solution:

We have, $\arg\left[\frac{(1+i\sqrt{3})(-\sqrt{3}-i)}{(1-i)(-i)}\right]$

$$
\begin{aligned}
Let\ \ z &= \frac{(1+i\sqrt{3})(-\sqrt{3}-i)}{(1-i)(-i)} \\
&= \frac{\left(i+i^2\sqrt{3}\right)(-\sqrt{3}-i)}{(1-i)\left(-i^2\right)} \\
&= \frac{(-\sqrt{3}+i)(-\sqrt{3}-i)}{(1-i)} \\
&= \frac{(-\sqrt{3})^2-i^2}{1-i} = \frac{3+1}{1-i} = \left(\frac{4}{1-i}\right) \\
\frac{4(1+i)}{(1-i)(1+i)} &= 4 \times \frac{1+i}{2}
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow \quad z = 2(1+i) \\
\therefore \arg(z) &= \arg(2+2i) = \tan^{-1}(1) \\
&= \tan^{-1}\left(\tan\frac{\pi}{4}\right) \\
&\Rightarrow \quad \arg(z) = \frac{\pi}{4}
\end{aligned}
$$

Question33

If $P(x,y)$ represents the complex number $z = x+iy$ in the argand plane and $\arg\left(\frac{z-3i}{z+4}\right) = \frac{\pi}{2}$, then the equation of the locus of P is

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Options:

A. $x^2+y^2+4x-3y=0$ and $3x-4y>0$

B. $x^2+y^2+4x-3y+2=0$ and $3x-4y>0$

C. $x^2+y^2+4x-3y=0$ and $3x-4y<0$

D. $x^2+y^2+4x-3y+2=0$ and $3x-4y<0$

Answer: C

Solution:

Given, $\arg\left(\frac{z-3i}{z+4}\right)=\frac{\pi}{2}$ and $z=x+iy$

$$\text{Now, let } \omega=\frac{z-3i}{z+4}=\frac{x+iy-3i}{x+iy+4}$$
$$=\frac{x+(y-3)i}{(x+4)+iy}\times\frac{(x+4)-iy}{(x+4)-iy}$$
$$=\frac{x(x+4)-ixy+(y-3)(x+4)i+y(y-3)}{(x+4)^2+y^2}$$
$$\Rightarrow\frac{x(x+4)+y(y-3)}{(x+4)^2+y^2}+i\frac{(y-3)(x+4)-xy}{(x+4)^2+y^2}$$

Now,

$$\arg(\omega)=\tan^{-1}\left(\frac{(y-3)(x+4)-xy)}{x(x+4)+y(y-3)}\right)=\frac{\pi}{2}$$
$$\Rightarrow\frac{(y-3)(x+4)-xy}{x(x+4)+y(y-3)}=\tan\frac{\pi}{2}$$
$$\Rightarrow\frac{4y-3x-12}{x^2+y^2+4x-3y}=\infty$$

So, clearly

$x^2+y^2+4x-3y=0$

and $3x-4y<0$

Question34

If $\alpha_1,\alpha_2,\alpha_3,\alpha_4$ and α_5 are the roots of $x^5-5x^4+9x^3-9x^2+5x-1=0$, then $\frac{1}{\alpha_1^2}+\frac{1}{\alpha_2^2}+\frac{1}{\alpha_3^2}+\frac{1}{\alpha_4^2}+\frac{1}{\alpha_5^2}$ is equal to

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Options:

A. 15

B. $\frac{1}{7}$

C. 7

D. 12

Answer: C

Solution:

Given,

$\alpha_1,\alpha_2,\alpha_3,\alpha_4$ and α_5 are roots of the equation

$$x^5-5x^4+9x^3-9x^2+5x-1=0$$
$$\Rightarrow x^5-x^4-4x^4+4x^3+5x^3-5x^2-4x^2+4x+x-1=0$$
$$\Rightarrow x^4(x-1)-4x^3(x-1)+5x^2(x-1)-4x(x-1)+1(x-1)=0$$
$$\Rightarrow(x-1)\left[x^4-4x^3+5x^2-4x+1\right]=0$$
$$\Rightarrow(x-1)\left(x^2-3x+1\right)\left(x^2-x+1\right)=0$$
$$\Rightarrow x=1,\frac{3+\sqrt{5}}{2},\frac{3-\sqrt{5}}{2},\frac{1+\sqrt{3}i}{2}$$

and $\frac{1-\sqrt{3}i}{2}$

$$\alpha_1=1\Rightarrow\alpha_1^2=1-\frac{1}{\alpha_1^2}=1$$
$$\alpha_2=\frac{3+\sqrt{5}}{2}\Rightarrow\alpha_2^2=\frac{(3+\sqrt{5})^2}{4}$$
$$\Rightarrow\quad\frac{1}{\alpha_2^2}=\frac{4}{(3+\sqrt{5})^2}$$
$$\Rightarrow\quad\alpha_3=\frac{3-\sqrt{5}}{2}\Rightarrow\alpha_3^2=\frac{(3-\sqrt{5})^2}{4}$$
$$\Rightarrow\quad\frac{1}{\alpha_3^2}=\frac{4}{(3-\sqrt{5})^2}$$
$$\alpha_4=\frac{1+\sqrt{3}i}{2}$$
$$\Rightarrow\alpha_4^2=\frac{(1+\sqrt{3}i)^2}{4}$$
$$\Rightarrow\frac{1}{\alpha_4^2}=\frac{4}{(1+\sqrt{3}i)^2}$$
$$\alpha_5=\frac{1-\sqrt{3}i}{2}\Rightarrow\alpha_5^2=\frac{(1-\sqrt{3}i)^2}{4}$$
$$\Rightarrow\quad\frac{1}{\alpha_5^2}=\frac{4}{(1-\sqrt{3}i)^2}$$
$$So,\quad\frac{1}{\alpha_1^2}+\frac{1}{\alpha_2^2}+\frac{1}{\alpha_3^2}+\frac{1}{\alpha_4^2}+\frac{1}{\alpha_5^2}$$
$$=1+\frac{4}{(3+\sqrt{5})^2}+\frac{4}{(3-\sqrt{5})^2}+\frac{4}{(1+\sqrt{3}i)^2}+\frac{4}{(1-\sqrt{3}i)^2}$$
$$=1+4\left[\frac{3^2+(\sqrt{5})^2-6\sqrt{5}+3^2+(\sqrt{5})^2+6\sqrt{5}}{(9-5)^2}+\frac{1-3-2\sqrt{3}i+1-3+2\sqrt{3}i}{(1+3)^2}\right]$$
$$=1+4\left[\frac{28}{16}+\frac{-4}{16}\right]$$
$$=1+4\left(\frac{28-4}{16}\right)=1+4\times\frac{24}{16}=7$$

Question35

If Z is a complex number such that $|Z|\leq 3$ and $\frac{-\pi}{2}\leq\operatorname{amp}Z\leq\frac{\pi}{2}$, then the area of the region formed by locus of Z is

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Options:

A. 9π

B. $\frac{9\pi}{2}$

C. 3π

D. $\frac{9\pi}{4}$

Answer: B

Solution:

$|Z| \leq 3 \Rightarrow$ Complex number Z lies within a circle centered at the origin with radius 3 .

$\frac{-\pi}{2} \leq \arg(z) \leq \frac{\pi}{2}$. This restricts z to the right half of the complex plane.

The intersection of these region is a semicircle with radius 3 . The area of semicircle is given by half the area of the full circle

The area of circle with radius r is $= \pi r^2$

$$= \pi(3)^2 = 9\pi$$

Half of this area $= \frac{9\pi}{2}$

So, area of region $= \frac{9\pi}{2}$ sq units.

Question36

The locus of the complex number Z such that $\arg\left(\frac{Z-1}{Z+1}\right) = \frac{\pi}{4}$ is

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Options:

A. a straight line

B. a circle

C. a parabola

D. an ellipse

Answer: B

Solution:

To determine the locus of the complex number Z, where $Z = x + iy$ (with x and y as real numbers), we analyze the given condition:

$\arg\left(\frac{Z-1}{Z+1}\right) = \frac{\pi}{4}$

This equation implies that the complex number $A = \frac{Z-1}{Z+1}$ forms an angle of $\frac{\pi}{4}$ with the positive real axis. Therefore, A lies on a line making an angle of $\frac{\pi}{4}$.

To simplify, we express:

$\frac{Z-1}{Z+1} = e^{i\frac{\pi}{4}} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

Thus, we have:

$\left|\frac{Z-1}{Z+1}\right| = \left|\frac{1}{\sqrt{2}} + i \cdot \frac{1}{\sqrt{2}}\right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$

This calculation indicates that the complex number $\frac{Z-1}{Z+1}$ is on the unit circle. Therefore, we equate:

$\left|\frac{Z-1}{Z+1}\right| = 1 \Rightarrow |Z-1| = |Z+1|$

This describes the locus as the perpendicular bisector of the line segment joining 1 and -1, which corresponds to the imaginary axis $x = 0$. Thus, the locus is a straight line on the imaginary axis.

Question37

All the values of $(8i)^{\frac{1}{3}}$ are

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Options:

A. $\pm(\sqrt{3}+i), -2i$

B. $\pm\sqrt{3}+i, -2i$

C. $\pm(\sqrt{3}-i), 2i$

D. $\pm(2+i), i$

Answer: B

Solution:

To find the values of $(8i)^{\frac{1}{3}}$, we utilize De Moivre's theorem. Express $8i$ in polar form as $8e^{i\left(\frac{\pi}{2}+2k\pi\right)}$, then take the cube root:

$(8i)^{\frac{1}{3}} = 8^{\frac{1}{3}} e^{i\left(\frac{\theta+2k\pi}{3}\right)} = 2e^{i\left(\frac{\pi}{6}+\frac{2k\pi}{3}\right)}$

Next, calculate this for different values of k:

For $k = 0$:

$2e^{i\left(\frac{\pi}{6}\right)} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) = 2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \sqrt{3} + i$

For $k = 1$:

$2e^{i\left(\frac{\pi}{6}+\frac{2\pi}{3}\right)} = 2e^{i\left(\frac{5\pi}{6}\right)} = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = -\sqrt{3} + i$

For $k = 2$:

$2e^{i\left(\frac{\pi}{6}+\frac{4\pi}{3}\right)} = 2e^{i\left(\frac{3\pi}{2}\right)} = 2(0-i) = -2i$

Therefore, all the values of $(8i)^{\frac{1}{3}}$ are $\sqrt{3}+i$, $-\sqrt{3}+i$, and $-2i$, or in other words, $\pm\sqrt{3}+i, -2i$.

Question38

If the number of real roots of $x^9 - x^5 + x^4 - 1 = 0$ is n, the number of complex roots having argument on imaginary axis is m and the number of complex roots having argument in 2nd quadrant is $K, m \cdot n \cdot k =$

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Options:

A. 6

B. 9

C. 12

D. 24

Answer: A

Solution:

$x^9 - x^5 + x^4 - 1 = 0$

$$\begin{aligned} \Rightarrow \quad x^5\left(x^4-1\right)+1\left(x^4-1\right) &= 0 \\ \Rightarrow \quad \left(x^5+1\right)\left(x^4-1\right) &= 0 \\ \Rightarrow \quad x^5+1=0 \text{ or } x^4-1=0 \\ \Rightarrow \quad x^5=-1 \text{ or } x^4=1 \\ \Rightarrow \quad x^5=-1 \text{ or } x=1,-1,i,-i \end{aligned}$$

$\therefore$ Number of real roots $(n) = 3$

Number of complex roots $(m) = 2$

Number of complex roots having argument in 2nd quadrant $(k) = 1$

$\therefore m \cdot n \cdot k = 3(2)(1) = 6$

Question39

Imaginary part of $\frac{(1-i)^3}{(2-i)(3-2i)}$ is

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Options:

A. $\frac{22}{65}$

B. $\frac{6}{65}$

C. $-\frac{6}{65}$

D. $-\frac{22}{65}$

Answer: D

Solution:

We have,

$$\frac{(1-i)^3}{(2-i)(3-2i)} = \frac{1^3 - i^3 - 3.1.i(1-i)}{(6-4i-3i+2i^2)}$$
$$[\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)]$$
$$= \frac{1-(-i)-3i+3i^2}{(6-7i-2)} = \frac{1+i-3i-3}{4-7i}$$
$$= \frac{-2-2i}{4-7i}$$
$$= \frac{-2-2i}{4-7i} \times \frac{4+7i}{4+7i} \qquad [rationalising]$$
$$= \frac{-(2+2i)(4+7i)}{4^2-(7i)^2}$$
$$= \frac{-(8+14i+8i-14)}{16+49} = \frac{6-22i}{65}$$

On comparing with $Z = x + iy$, we get

Imaginary part is $\frac{-22}{65}$.

Question40

The square root of $7 + 24i$

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Options:

A. $4 - 3i$

B. $3 + 4i$

C. $3 - 4i$

D. $4 + 3i$

Answer: D

Solution:

To find the square root of $7 + 24i$, set it equal to $x + yi$, where $x, y \in \mathbb{R}$.

Squaring both sides gives:

$(x + yi)^2 = x^2 - y^2 + 2xyi$

This leads to:

$x^2 - y^2 + 2xyi = 7 + 24i$

By equating the real and imaginary parts, we obtain:

$x^2 - y^2 = 7 \quad \text{(equation 1)}$

$2xy = 24 \quad \Rightarrow \quad xy = 12 \quad \text{(equation 2)}$

From equation 2, solve for y:

$y = \frac{12}{x}$

Substitute $y = \frac{12}{x}$ into equation 1:

$x^2 - \left(\frac{12}{x}\right)^2 = 7$

Simplifying the equation, we get:

$x^2 - \frac{144}{x^2} = 7$

$x^4 - 144 = 7x^2$

$x^4 - 7x^2 - 144 = 0$

This can be factored as:

$(x^2 - 16)(x^2 + 9) = 0$

Since x is real, we discard $x^2 + 9 = 0$ as it does not lead to real solutions for x. Therefore:

$x^2 = 16 \quad \Rightarrow \quad x = \pm 4$

For $x = 4$, $y = 3$.

For $x = -4$, $y = -3$.

Thus, the square roots of $7 + 24i$ are $\pm(4 + 3i)$.

Therefore, the correct solution is $4 + 3i$.

Question41

If n is an integer and $Z = \cos\theta + i\sin\theta, \theta \neq (2n+1)\frac{\pi}{2}$, then $\frac{1+Z^{2n}}{1-Z^{2n}} =$

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Options:

A. $i \tan n\theta$

B. $i \cot n\theta$

C. $-i \tan n\theta$

D. $-i \cot n\theta$

Answer: D

Solution:

We have,

$Z = \cos\theta + i\sin\theta, \theta \neq (2n+1)\frac{\pi}{2}$

Now, $\frac{1+Z^{2n}}{1-Z^{2n}} = \frac{1+(\cos\theta+i\sin\theta)^{2n}}{1-(\cos\theta+i\sin\theta)^{2n}}$

$= \frac{1+(\cos 2n\theta+i\sin 2n\theta)}{1-(\cos 2n\theta+i\sin 2n\theta)}$

[By De Moivre's theorem]

$$= \frac{1 + (2\cos^2 n\theta - 1) + 2i \sin n\theta \cos n\theta}{1 - (1 - 2\sin^2 n\theta) - 2i \sin n\theta \cos n\theta}$$

$$[\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A]$$

$$= \frac{2\cos n\theta[\cos n\theta + i \sin n\theta]}{2\sin n\theta[\sin n\theta - i\cos n\theta]}$$

$$= -i \cot n\theta \left[\frac{\cos n\theta + i \sin n\theta}{\cos n\theta + i \sin n\theta}\right]$$

$$= -i \cot n\theta.$$

Question42

The complex conjugate of $(4 - 3i)(2 + 3i)(1 + 4i)$ is.

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Options:

A. $7 + 74i$

B. $-7 + 74i$

C. $-7 - 74i$

D. $7 - 74i$

Answer: C

Solution:

When calculating the complex conjugate of the expression $(4 - 3i)(2 + 3i)(1 + 4i)$, follow these steps:

First, compute the product:

$(4 - 3i)(2 + 3i) = 8 + 12i - 6i - 9i^2 = 8 + 6i + 9 = 17 + 6i$

Here, we used the fact that $i^2 = -1$.

Next, multiply this result by $(1 + 4i)$:

$(17 + 6i)(1 + 4i) = 17 + 68i + 6i + 24i^2 = 17 + 74i - 24$

Simplifying this, we get:

$-7 + 74i$

To find the complex conjugate of $-7 + 74i$, change the sign of the imaginary part:

$-7 - 74i$

So, the complex conjugate of the expression is $-7 - 74i$.

Question43

If the amplitude of $(z - 2)$ is $\frac{\pi}{2}$, then the locus of z is

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Options:

A. $x = 0, y > 0$

B. $x = 2, y > 0$

C. $x > 0, y = 2$

D. $x > 0, y = 0$

Answer: B

Solution:

To determine the locus of z, given that the amplitude (also known as the argument) of $(z - 2)$ is $\frac{\pi}{2}$, we proceed as follows:

Let $z = x + iy$, where x and y are real numbers. Given the condition:

$\arg(z - 2) = \frac{\pi}{2}$

Substituting $z = x + iy$ into the expression, we have:

$\arg((x - 2) + iy) = \frac{\pi}{2}$

This implies:

$\tan^{-1}\left(\frac{y}{x-2}\right) = \frac{\pi}{2}$

From the property of arctangent, we know:

$\frac{y}{x-2} \rightarrow \infty$

This scenario occurs when $x - 2 = 0$. Hence,

$x = 2$

Thus, the locus of z is a vertical line at $x = 2$, which is parallel to the Y-axis, only considered for $y > 0$.

Therefore, the locus of z is a vertical line $x = 2$ for $y > 0$.

Question44

If ω is the cube root of unity, $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} + \frac{a+b\omega+c\omega^2}{b+c\omega+b\omega^2} =$

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Options:

A. 2

B. -2

C. 1

D. -1

Answer: D

Solution:

If ω is the cube root of unity.

$\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}+\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}=?$

Now, we write as

$$A=a+b\omega+c\omega^2$$
$$B=b+c\omega+a\omega^2$$
$$C=c+a\omega+b\omega^2$$

Find common denominator for the fractions.

$\frac{A}{C}+\frac{A}{B}=\frac{A\cdot B+A\cdot C}{B\cdot C}=\frac{A(B+C)}{BC}$

Now, we know that according to given condition.

$$\omega^3=1$$
$$1+\omega+\omega^2=0$$
$$B+C=\left(b+c\omega+a\omega^2\right)+\left(c+a\omega+b\omega^2\right)$$
$$b+c\omega+a\omega^2+c+a\omega+b\omega^2$$
$$=\left(b+\omega^2b\right)+(c\omega+c)+\left(a\omega^2+a\omega\right)$$
$$=b\left(1+\omega^2\right)+c(1+\omega)+a\left(\omega^2+\omega\right)$$
$$\left(\because \text{ using } 1+w+w^2=0\right)$$
$$1+\omega^2=-\omega, 1+\omega=-\omega^2$$

$$B+C=b(-\omega)+C\left(-\omega^2\right)+a(-1)$$
$$=-a-b\omega-c\omega^2$$

Thus, $B+C=-A$

Because, $A=a+b\omega+c\omega^2$

Substituting this into the expression

$\frac{A(B+C)}{BC}=\frac{A(-A)}{BC}=\frac{-A^2}{BC}$

The denominator (BC) does not change the overall simplification that each term involves $\bar{A}, B$ and C with symmetrical properties.

Thus, $\frac{A}{C}+\frac{A}{B}=-1$

Question45

If $(3+i)$ is a root of $x^2+ax+b=0$, then $a=$

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Options:

A. 3

B. -3

C. 6

D. -6

Answer: D

Solution:

Given, $(3+i)$ is a root of
$x^2+ax+b=0$
$(3+i)^2+a(3+i)+b=0$
$9-1+6i+a(3+i)+b=0$
$8+6i+3a+ai+b=0$
$(8+3a+b)+i(6+a)=0$
$3a+b=-8$
$a+6=0 \Rightarrow a=-6$

Question46

If $z_1=10+6i, z_2=4+6i$ and z is any complex number such that the argument of $\frac{(z-z_1)}{(z-z_2)}$ is $\frac{\pi}{4}$,

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Options:

A. $|z-7-9i|=3\sqrt{2}$

B. $|z-7-9i|=2\sqrt{2}$

C. $|z-3+9i|=3\sqrt{2}$

D. $|z+3-9i|=2\sqrt{2}$

Answer: A

Solution:

Given, $z_1=10+6i, z_2=4+6i$

and $\arg\left(\frac{z-z_1}{z-z_2}\right)=\frac{\pi}{4}$

Let $z=x+iy$

Now, $\frac{z-z_1}{z-z_2}=\frac{(x+iy)-(10+6i)}{(x+iy)-(4+6i)}$

$$=\frac{(x-10)+i(y-6)}{(x-4)+i(y-6)}$$
$$=\frac{(x-10)+i(y-6)}{(x-4)+i(y-6)}\times\frac{(x-4)-i(y-6)}{(x-4)-i(y-6)}$$
$$=\frac{(x-10)(x-4)+(y-6)^2+i[-(x-10)(y-6)+(y-6)(x-4)]}{(x-4)^2+(y-6)^2}$$

Since, $\arg\left(\frac{z-z_1}{z-z_2}\right)=\frac{\pi}{4}$

i.e. $\tan\theta=1$

$\therefore \frac{(y-6)(x-4)-(x-10)(y-6)}{(x-10)(x-4)+(y-6)^2}=1$

$\Rightarrow \frac{xy-6x-4y+24-xy+6x+10y-60}{x^2-14x+40+y^2-12y+36}=1$

$\Rightarrow 6y-36=x^2+y^2-12y-14x+76$

$\Rightarrow x^2+y^2-18y-14x+112=0$

$\Rightarrow \quad (x-7)^2+(y-9)^2=18$

$\Rightarrow |z-(7+9i)|=3\sqrt{2}$

Question47

If $\frac{3-2i\sin\theta}{1+2i\sin\theta}$ is purely imaginary number, then $\theta=$

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Options:

A. $2n\pi \pm \frac{\pi}{4}$

B. $2n\pi \pm \frac{\pi}{2}$

C. $n\pi \pm \frac{\pi}{3}$

D. $n\pi \pm \frac{\pi}{6}$

Answer: C

Solution:

$\frac{3-2i\sin\theta}{1+2i\sin\theta}=\frac{(3-2i\sin\theta)}{(1+2i\sin\theta)}\times\frac{(1-2i\sin\theta)}{(1-2i\sin\theta)}$

(by rationalising)

$$=\frac{3-6i\sin\theta-2i\sin\theta-4\sin^2\theta}{1+4\sin^2\theta}$$
$$=\frac{3-4\sin^2\theta-8i\sin\theta}{1+4\sin^2\theta}$$

Since, $\frac{3-2i\sin\theta}{1+2i\sin\theta}$ is purely imaginary.

$\therefore \quad \frac{3-4\sin^2\theta}{1+4\sin^2\theta}=0$

$\Rightarrow \quad 3-4\sin^2\theta=0 \Rightarrow 4\sin^2\theta=3$

$\Rightarrow \quad \sin^2\theta=\frac{3}{4} \Rightarrow \sin\theta=\frac{\sqrt{3}}{2}$

$\Rightarrow \quad \theta=n\pi\pm\frac{\pi}{3}$

Question48

If $z=x+iy, x^2+y^2=1$ and $z_1=ze^{i\theta}$, then $\frac{z_1^{2n}-1}{z_1^{2n}+1}=$

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Options:

A. $-i\tan\left(n\left(\theta+\tan^{-1}\left(\frac{y}{x}\right)\right)\right)$

B. $i\cot\left(n\left(\theta+\tan^{-1}\frac{y}{x}\right)\right)$

C. $i \tan\left(n\left(\theta+\tan^{-1}\frac{x}{u}\right)\right)$

D. $i \tan\left(n\left(\theta+\tan^{-1}\frac{y}{x}\right)\right)$

Answer: D

Solution:

Given, $z=x+iy, x^2+y^2=1$ and

$z_1=ze^{i\theta}$

$z_1=(x+iy)e^{i\theta}$

Taking (2n)th power of both sides of the equation

$$\begin{aligned}
z_1^{2n} &= (x+iy)^{2n}e^{i2n\theta} \\
&= r^{2n}(\cos(2n\theta)+i\sin(2n\theta)) \\
z_1^{2n}+1 &= r^{2n}(\cos(2n\theta)+i\sin(2n\theta))+1 \\
&= (\cos(2n\theta)+i\sin(2n\theta))+1 \\
&[\because x^2+y^2=1] \\
z_1^{2n}-1 &= (\cos(2n\theta)+i\sin(2n\theta))-1 \\
\frac{z_1^{2n}-1}{z_1^{2n}+1} &= \frac{(\cos(2n\theta)+i\sin(2n\theta))-1}{(\cos(2n\theta)+i\sin(2n\theta))+1} \\
&= \frac{1-2\sin^2(n\theta)+i\sin(2n\theta)-1}{2\cos^2(n\theta)-1+i\sin(2n\theta)+1} \\
&= \frac{-2\sin^2(n\theta)+2i\sin(n\theta)\cos(n\theta)}{2\cos^2(n\theta)+2i\sin(n\theta)\cos(n\theta)} \\
&= \frac{[i\sin(n\theta)+\cos(n\theta)]i\sin(n\theta)}{[\cos(n\theta)+i\sin(n\theta)]\cos(n\theta)} \\
&= i\tan n\theta = i\tan\left(n\left(\theta+\tan^{-1}\frac{y}{x}\right)\right)
\end{aligned}$$

Question49

If the point P represents the complex number $z=x+iy$ in the argand plane and if $\frac{z+i}{z-i}$ is a purely imaginary number, then the locus of P is

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Options:

A. $x^2+y^2+x-y=0$ and $(x,y)\neq(1,0)$

B. $x^2+y^2-x+y=0$ and $(x,y)\neq(1,0)$

C. $x^2+y^2-x+y=0$ and $(x,y)=(1,0)$

D. $x^2+y^2+x+y=0$

Answer: B

Solution:

We have, $\frac{z+i}{z-1}=\frac{x+iy+i}{x+iy-1}$

$$= \frac{x + i(1+y)}{(x-1)+iy} \times \frac{(x-1)-iy}{(x-1)-iy}$$
$$= \frac{x(x-1) + i(1+y)(x-1) - ixy + y(1+y)}{(x-1)^2 + y^2}$$
$$= \frac{x^2 - x + y^2 + i(x + xy - y - 1 - xy)}{(x-1)^2 + y^2}$$
$$= \frac{x^2 + y^2 - x + y}{(x-1)^2 + y^2} + i \frac{(x - y - 1)}{(x-1)^2 + y^2}$$

Since, $\frac{z+i}{z-1}$ is purely imaginary.

So, $\frac{x^2+y^2-x+y}{(x-1)^2+y^2} = 0$

$x^2 + y^2 - x + y = 0$

Where, $x \neq 1, y \neq 0$

Question50

$S = \{z \in C / |z + 1 - i| = 1\}$ represents

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Options:

A. the circle with centre at $(-1, 1)$ and radius 1 unit

B. the circle with cente at $(1, -1)$ and radius 1 unit

C. the closed circular disc with centre at $(1, -1)$ and radius 1 unt

D. the closed circular disc with centre at($-1, 1$) and radius 1 unt

Answer: A

Solution:

The given set $S = \{z \in \mathbb{C} \mid |z + 1 - i| = 1\}$ can be analyzed as follows:

First, consider the expression $|z + 1 - i| = 1$.

Break down z in terms of real and imaginary components:

$z = x + iy,$

where x and y are real numbers. Thus, we can substitute z in the expression:

$|x + iy + 1 - i| = 1$

Simplifying inside the absolute value, we have:

$|(x+1) + i(y-1)| = 1$

The expression $|(x+1) + i(y-1)|$ represents the distance from the point $(-1, 1)$ in the complex plane. Therefore, the set of all such points for which this distance is 1 forms a circle centered at $(-1, 1)$ with a radius of 1. Mathematically, this can be simplified to:

$\sqrt{(x+1)^2 + (y-1)^2} = 1$

Squaring both sides, we get:

$(x+1)^2 + (y-1)^2 = 1$

Thus, S represents a circle with center at $(-1,1)$ and radius 1 unit.

Question51

If m, n are respectively the least positive and greatest negative integer value of k such that $\left(\frac{1-i}{1+i}\right)^k = -i$, then $m - n =$

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Options:

A. 4

B. 0

C. 6

D. 2

Answer: A

Solution:

To solve for the least positive and greatest negative integer values of k in the expression $\left(\frac{1-i}{1+i}\right)^k = -i$, follow these steps:

Start with the given equation:

$\left(\frac{1-i}{1+i}\right)^k = -i$

Simplify the fraction $\frac{1-i}{1+i}$:

$\frac{1-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{(1-i)^2}{(1+i)(1-i)}$

Calculate the numerator and the denominator:

$(1-i)^2 = 1 - 2i + i^2 = 1 - 2i - 1 = -2i$

$(1+i)(1-i) = 1 + i - i - i^2 = 1 + 1 = 2$

Substitute back into the fraction:

$\frac{(1-i)^2}{(1+i)(1-i)} = \frac{-2i}{2} = -i$

Therefore, the expression simplifies to:

$(-i)^k = -i$

For $(-i)^k = -i$, it implies:

$k \equiv 1 \pmod{4}$

From this, determine the least positive integer m and greatest negative integer n:

$m = 1$ (since $k = 1$ is the smallest positive integer solution)

$n = -3$ (since $k = -3$ satisfies the condition $-i$ in the sequence and is the largest negative solution)

Calculate $m - n$:

$m - n = 1 - (-3) = 4$

Thus, $m-n=4$.

Question52

If a complex number z is such that $\frac{z-2i}{z-2}$ is purely imaginary number and the locus of z is a closed curve, then the area of the region bounded by that closed curve and lying in the first quadrant is $\frac{z-2i}{z-2}$

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Options:

A. 2π

B. $\frac{\pi}{2}$

C. π

D. $\frac{\pi}{4}$

Answer: A

Solution:

$\frac{z-2i}{z-2}$ is purely imaginary

Let $z=x+iy$

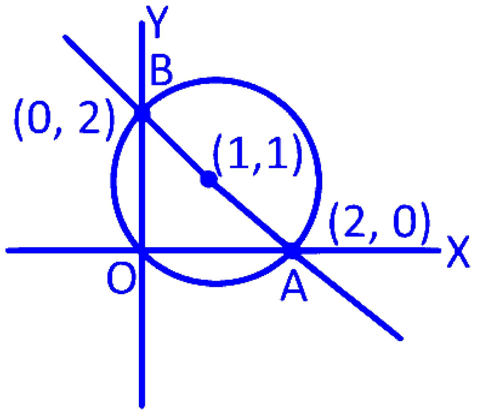
$\therefore \frac{x+iy-2i}{x+iy-2}=\frac{x+i(y-2)}{(x-2)+iy}\times\frac{(x-2)-iy}{(x-2)-iy}$

Real part of $\frac{z-2i}{z-2}=0$

$x(x-2)+y(y-2)=0$

$\Rightarrow \quad x^2+y^2-2x-2y=0$

$\Rightarrow (x-1)^2+(y-1)^2=2$, which is equation of circle with radius $\sqrt{2}$ units.



Required area $=\frac{1}{2}\times 2\times 2+\frac{\pi}{2}(\sqrt{2})^2=(2+\pi)$ sq units

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Question53

Real part of $\frac{(\cos a+i\sin a)^6}{(\sin b+i\cos b)^8}$ is

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Options:

A. $\sin(6a-8b)$

B. $\cos(6a-8b)$

C. $\sin(6a+8b)$

D. $\cos(6a+8b)$

Answer: D

Solution:

We have, $\frac{(\cos a+i\sin a)^6}{(\sin b+i\cos b)^8}$

$$=\frac{(\cos a+i\sin a)^6}{(-i^2\sin b+i\cos b)^8} \quad [\because 1=-i^2]$$

$$=\frac{(\cos a+i\sin a)^6}{i^8(\cos b-i\sin b)^8}$$

$$=\frac{(\cos a+i\sin a)^6}{(\cos b-i\sin b)^8}$$

$$=\frac{\cos 6a+i\sin 6a}{\cos 8b-i\sin 8b}$$

$$=\frac{(\cos 6a+i\sin 6a)(\cos 8b+i\sin 8b)}{(\cos 8b-i\sin 8b)(\cos 8b+i\sin 8b)}$$

$\therefore$ Required real part

$$=\frac{\cos 6a\cos 8b-\sin^8 6a\sin 8b}{\cos^2 8b+\sin^2 8b}$$

$$=\cos(6a+8b)$$

Question54

If real parts of $\sqrt{-5-12i}$, $\sqrt{5+12i}$ are positive values, the real part of $\sqrt{-8-6i}$ is a negative value and $a+ib=\frac{\sqrt{-5-12i}+\sqrt{5+12i}}{\sqrt{-8-6i}}$, then $2a+b=$

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Options:

A. 3

B. 2

C. -3

D. -2

Answer: C

Solution:

To find $a+ib=\frac{\sqrt{-5-12i}+\sqrt{5+12i}}{\sqrt{-8-6i}}$ and calculate $2a+b$:

Calculate $\sqrt{-5-12i}$ and $\sqrt{5+12i}$:

Since the real parts of $\sqrt{-5-12i}$ and $\sqrt{5+12i}$ are positive:

$\sqrt{-5-12i}=\sqrt{\frac{13-5}{2}}+i\sqrt{\frac{13+5}{2}}=2-3i$

$\sqrt{5+12i}=\sqrt{\frac{13+5}{2}}+i\sqrt{\frac{13-5}{2}}=3+2i$

Calculate $\sqrt{-8-6i}$:

Given that the real part is negative:

$\sqrt{-8-6i}=-\sqrt{\frac{10-8}{2}}+i\sqrt{\frac{10+8}{2}}=-1+3i$

Find $a+ib$:

Using the calculations from the previous steps:

$a+ib=\frac{(2-3i)+(3+2i)}{-1+3i}=\frac{5-i}{-1+3i}$

Multiply the numerator and the denominator by the conjugate of the denominator:

$=\frac{(5-i)(-1-3i)}{(-1+3i)(-1-3i)}$

Simplify:

$=\frac{-5+i-15i-3}{1+9}=\frac{-8-14i}{10}$

$=-\frac{4}{5}-\frac{7i}{5}$

Thus, $a=-\frac{4}{5}$ and $b=-\frac{7}{5}$.

Calculate $2a+b$:

$2a+b=2\left(-\frac{4}{5}\right)+\left(-\frac{7}{5}\right)=-\frac{8}{5}-\frac{7}{5}=-3$

Hence, the result is -3.

Question55

The set of all real values of c for which the equation $z\overline{z}+(4-3i)z+(4+3i)\overline{z}+c=0$ represents a circle, is

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Options:

A. [25, 50]

B. [-5, 5]

C. $[-20, -5] \cup [5, 20]$

D. [-25]

Answer: D

Solution:

The given equation $z\bar{z} + (4-3i)z + (4+3i)\bar{z} + c = 0$ is analyzed to determine the real values of c that form a circle.

Key Steps

Identifying the Circle's Center:

$z\bar{z}$ is the modulus squared of z, and the terms $(4-3i)z$ and $(4+3i)\bar{z}$ suggest the transformation involves the complex number $4-3i$.

Thus, the center of the circle is at $4-3i$.

Finding the Radius:

To find the radius, use the formula that incorporates the modulus of the center:

$\sqrt{(4)^2 + (-3)^2 - c} = \sqrt{25 - c}$

The radius needs to be non-negative for the equation to represent a circle:

$\sqrt{25 - c} \geq 0$

which simplifies to:

$25 - c \geq 0 \quad \Rightarrow \quad c \leq 25$

Conclusion

The real values of c for which the equation represents a circle are in the interval:

$(-\infty, 25]$

Question56

If $z = x + iy$ is a complex number, then the number of distinct solutions of the equation $z^3 + \bar{z} = 0$ is

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Options:

A. 1

B. 3

C. Infinite

D. 5

Answer: D

Solution:

Given the equation $z^3 + \bar{z} = 0$, where $z = x + iy$ is a complex number, we can find the distinct solutions as follows:

Rewrite the Equation:

Start with:

$z^3 + \overline{z} = 0$

This implies:

$z^3 = -\overline{z}$

Equate Magnitudes:

Taking the magnitude of both sides, we have:

$|z^3| = |-\overline{z}|$

This simplifies to:

$|z|^3 = |\overline{z}|$

Since $|\overline{z}| = |z|$, the equation becomes:

$|z|^3 = |z|$

Solve for Magnitude:

This equation leads to:

$|z|(|z|^2 - 1) = 0$

Therefore, either $|z| = 0$ or $|z|^2 = 1$.

Consider Each Case:

If $|z| = 0$, then $z = 0$. This gives one distinct solution: $z = 0$.

If $|z|^2 = 1$, then $|z| = 1$ and consequently $z\overline{z} = 1$. This implies:

$z = \frac{1}{z}$

Substitute Back:

Substituting $\overline{z} = \frac{1}{z}$ into the original equation:

$z^3 + \frac{1}{z} = 0$

Multiply through by z to clear the fraction:

$z^4 + 1 = 0$

Solve $z^4 + 1 = 0$:

This equation has four roots:

$z = e^{i(\pi/4)}, e^{i(3\pi/4)}, e^{i(5\pi/4)}, e^{i(7\pi/4)}$

Count Distinct Solutions:

Including $z = 0$, we have:

One solution for $z = 0$

Four solutions from $z^4 + 1 = 0$

Therefore, the number of distinct solutions to the equation is $\boxed{5}$.

Question57

By simplifying $i^{18} - 3i^7 + i^2\left(1 + i^4\right)(i)^{22}$, we get

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Options:

A. $-1+3i$

B. $1-3i$

C. $1+3i$

D. $-1-3i$

Answer: C

Solution:

$i^{18}-3i^7+i^2\left(1+i^4\right)(i)^{22}$
$=i^{4\times4+2}-3i^{4+3}+-1(1+1)\left(i^2\right)^{11}$
$=i^2-3i^3+2 \quad [\because i^2=-1 \text{ and } i^4=1]$
$=-1+3i+2=1+3i$

Question58

The values of x for which $\sin x+i\cos 2x$ and $\cos x-i\sin 2x$ are conjugate to each other are

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Options:

A. $x=n\pi\pm\frac{\pi}{6}$

B. None

C. $x=n\pi\pm\frac{\pi}{3}$

D. $x=\left(n+\frac{1}{2}\right)\pi$

Answer: B

Solution:

Conjugate of $\cos x-i\sin 2x$ is $\cos x+i\sin 2x$

So, $\sin x+i\cos 2x=\cos x+i\sin 2x$

Comparing real and imaginary parts,

$\sin x=\cos x$ or $\tan x=1$

and $\cos 2x=\sin 2x$ or $\tan 2x=1$

But for same value of x, both $\tan x$ and $\tan 2x$ can not.Hence, no solution is possible.

Question59

The locus of a point z satisfying $|z|^2 = \mathrm{Re}(z)$ is a circle with centre

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Options:

A. $\left(0, \frac{1}{2}\right)$

B. $\left(-\frac{1}{2}, 0\right)$

C. $\left(\frac{1}{2}, 0\right)$

D. $\left(0, -\frac{1}{2}\right)$

Answer: C

Solution:

Let $z = x + iy$

$|z| = \sqrt{x^2 + y^2}$

Now, $|z|^2 = \mathrm{Re}(z)$

$$x^2 + y^2 = x$$
$$x^2 + y^2 - x = 0$$
$$g = 1/2, f = 0$$

So, centre of circle $\left(\frac{1}{2}, 0\right)$.

Question60

Multiplicative inverse of the complex number $(\sin\theta, \cos\theta)$ is

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Options:

A. $(\sin\theta, \cos\theta)$

B. $(\sin\theta, -\cos\theta)$

C. $(\cos\theta, -\sin\theta)$

D. $(-\cos\theta, \sin\theta)$

Answer: B

Solution:

Multiplicative inverse of complex number $(\sin\theta, \cos\theta)$ is

$$\begin{aligned} z &= \sin\theta + i\cos\theta \\ \Rightarrow \frac{1}{z} &= \frac{(\sin\theta - i\cos\theta)}{(\sin\theta + i\cos\theta)(\sin\theta - i\cos\theta)} \\ &= \frac{(\sin\theta - i\cos\theta)}{\sin^2\theta + \cos^2\theta} = (\sin\theta - i\cos\theta) \end{aligned}$$

Thus, multiplicative inverse of $(\sin\theta, \cos\theta)$ is $(\sin\theta, -\cos\theta)$.

Question61

$$\sum_{k=0}^{440} i^k = x + iy \Rightarrow x^{100} + x^{99}y + x^{242}y^2 + x^{97}y^3 =$$

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Options:

A. 0

B. -4

C. 4

D. 1

Answer: D

Solution:

$$\begin{aligned} &\sum_{k=0}^{440} i^k = x + iy \\ &\Rightarrow \quad x + iy = 1 + i + i^2 + i^3 + i^4 + \ldots. + i^{440} = -1 \\ &\therefore \quad x = -1, y = 0 \\ &\text{Then, } x^{100} + x^{99} \cdot y + x^{242} \cdot y^2 + x^{97} \cdot y^3 \\ &\Rightarrow \quad (-1)^{100} + 0 = 1 \end{aligned}$$

Question62

If $e^{i\theta} = \operatorname{cis}\theta$, then $\sum_{n=0}^{\infty} \frac{\cos(n\theta)}{2^n} =$

Options:

A. $(4+2\cos\theta)/(5-4\cos\theta)$

B. $(4-2\cos\theta)/(5+4\cos\theta)$

C. $(4-2\cos\theta)/(5-4\cos\theta)$

D. $(4+2\cos\theta)/(5+4\cos\theta)$

Answer: C

Solution:

$$e^{i\theta}=\operatorname{cis}\theta=\cos\theta+i\sin\theta$$
$$\sum_{n=0}^{\infty}\frac{\cos(n\theta)}{2^n}$$
$$=\operatorname{Re}\left[\sum_{n=0}^{\infty}\frac{\cos n\theta}{2^n}\right]$$
$$=\operatorname{Re}\left[\sum_{n=0}^{\infty}\left(\frac{\cos\theta}{2}\right)^n\right] \text{ [Using De-moivre theorem]}$$
$$=\operatorname{Re}\left[\sum_{n=0}^{\infty}\left(\frac{e^{i\theta}}{2}\right)^n\right]$$
$$=\operatorname{Re}\left[1+\frac{e^{i\theta}}{2}+\left(\frac{e^{i\theta}}{2}\right)^2+\ldots.\infty\right]$$
$$1+\frac{e^{i\theta}}{2}+\left(\frac{e^{i\theta}}{2}\right)^2+\ldots.. \text{ is an G P}$$
$$=\operatorname{Re}\left[\frac{1}{1-\frac{e^{i\theta}}{2}}\right]=\operatorname{Re}\left[\frac{2}{2-e^{i\theta}}\right]$$
$$=\operatorname{Re}\left[\frac{2}{2-\cos\theta-i\sin\theta}\right]$$
$$=\operatorname{Re}\left[\frac{2(2-\cos\theta-i\sin\theta)}{(2-\cos\theta)^2-i^2\sin^2\theta}\right]$$
$$=\operatorname{Re}\left[\frac{4-2\cos\theta}{4+\cos^2\theta-4\cos\theta+\sin^2\theta}\right.$$
$$\left.+\frac{i\sin\theta}{4+\cos^2\theta-4\cos\theta+\sin^2\theta}\right]$$
$$=\operatorname{Re}\left[\frac{4-2\cos\theta}{5-4\cos\theta}+\frac{i\sin\theta}{5-4\cos\theta}\right]$$
$$=\frac{4-2\cos\theta}{5-4\cos\theta}$$

Question63

$$iz^3+z^2-z+i=0\Rightarrow|z|=$$

Options:

A. 1/2

B. 2

C. 3/2

D. 1

Answer: D

Solution:

$$\begin{aligned} & iz^3+z^2-z+i=0 \\ \Rightarrow\ & iz^3-z+z^2+i=0 \\ \Rightarrow\ & iz^3+i^2z+z^2+i=0 \\ \Rightarrow\ & iz\left(z^2+i\right)+\left(z^2+i\right)=0 \\ \Rightarrow\ & \left(z^2+i\right)(iz+1)=0 \\ \Rightarrow\ & \left(z^2+i\right)\left(iz-i^2\right)=0 \\ \Rightarrow\ & \left(z^2+i\right)i(z-i)=0 \\ \Rightarrow\ & z^2=-i \text{ or } z=i \\ & z^2=1\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right) \\ \text{or}\ & z=1\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right) \end{aligned}$$

We can see that all the roots have modulus as 1.

So, $|z|=1$

Question64

If $\frac{x-1}{3+i}+\frac{y-1}{3-i}=i$, then the true statement among the following is

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Options:

A. $x<0, y<0$

B. $x<0, y>0$

C. $x>0, y<0$

D. $x>0, y>0$

Answer: B

Solution:

Given

$$\frac{x-1}{3+i}+\frac{y-1}{3-i}=i \qquad (x, y \text{ are real})$$

Step 1: Rationalize the denominators (find reciprocals)

$$\frac{1}{3+i}=\frac{3-i}{(3+i)(3-i)}=\frac{3-i}{10}$$

$$\frac{1}{3-i}=\frac{3+i}{(3-i)(3+i)}=\frac{3+i}{10}$$

So the equation becomes:

$$(x-1)\frac{3-i}{10}+(y-1)\frac{3+i}{10}=i$$

Multiply by 10:

$$(x-1)(3-i)+(y-1)(3+i)=10i$$

Step 2: Expand and separate Real & Imaginary parts

$$(x-1)(3-i)=3(x-1)-i(x-1)$$

$$(y-1)(3+i)=3(y-1)+i(y-1)$$

Add them:

$$3(x-1)+3(y-1)+i\big(-(x-1)+(y-1)\big)=10i$$

✅ **Real part:**

$$3(x-1)+3(y-1)=0 \Rightarrow 3(x+y-2)=0 \Rightarrow x+y=2$$

✅ **Imaginary part:**

$$-(x-1)+(y-1)=10 \Rightarrow -x+1+y-1=10 \Rightarrow y-x=10$$

Step 3: Solve the two equations

$$x+y=2$$

$$y-x=10$$

Add both:

$$2y=12 \Rightarrow y=6$$

Then

$$x=2-y=2-6=-4$$

Conclusion

$$x=-4<0, \quad y=6>0$$

✅ Correct Option: (B) $x<0,\ y>0$

Question65

The number of integer solutions of the equation $|1-i|^x=2^x$ is

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Options:

A. 1

B. 0

C. 2

D. 3

Answer: A

Solution:

$|1-i|^x = 2^x$

$\Rightarrow \quad \left(\sqrt{\Lambda)^2+(-1)^2}\right)^x = 2^x$

$\Rightarrow \quad (\sqrt{2})^x = 2^x$

$\Rightarrow \quad 2^{\frac{x}{2}} = 2^x$

$\Rightarrow \quad \frac{x}{2} = x$

$\Rightarrow \quad 0 = \frac{x}{2}$

$\Rightarrow \quad x = 0$

Number of integer solution is 1.

Question66

Let Z_1, Z_2 and Z_3 be three non zero complex numbers such that $a = |Z_1|, b = |Z_2|$ and $c = |Z_3|$, if the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$, then

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Options:

A. $|Z_1| = |Z_2| = |Z_3| = abc$

B. $|Z_1| + |Z_2| + |Z_3| = 0$

C. $|Z_1| + |Z_2| + |Z_3| = abc$

D. $|Z_1 - Z_2| = |Z_2 - Z_3|$

Answer: B

Solution:

$$\left|\begin{array}{lll}a & b & c \\ b & c & a \\ c & a & b\end{array}\right|=a\left(b c-a^2\right)-b\left(b^2-a c\right)+c\left(a b-c^2\right)$$
$$\begin{aligned} & \Rightarrow a b c-a^3-b^3+a b c+a b c-c^3=0 \\ & \Rightarrow 3 a b c-a^3-b^3-c^3=0 \\ & \Rightarrow a^3+b^3+c^3-3 a b c=0 \\ & \Rightarrow(a+b+c)=0\end{aligned}$$

Since, $a=\left|Z_1\right|, b=\left|Z_2\right|, c=\left|Z_3\right|$

$\therefore \quad\left|Z_1\right|+\left|Z_2\right|+\left|Z_3\right|=0$

Question67

If $\left|z_1+z_2\right|^2=\left|z_1\right|^2+\left|z_2\right|^2$, where z_1 and z_2 are two complex numbers, then

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Options:

A. $\frac{z_1}{z_2}$ is purely real

B. $\frac{z_1}{z_2}$ is purely imaginary

C. $\arg \left(\frac{z_1}{z_2}\right)=\frac{\pi}{4}$

D. $\left|\frac{z_1}{z_2}\right|=1$

Answer: B

Solution:

$\left|z_1+z_2\right|^2=\left|z_1\right|^2+\left|z_2\right|^2 \quad$ (given) ..... (i)

Let $z_1=x+i y, z_2=a+i b$.

Then

$$\begin{aligned} & \frac{z_1}{z_2}=\frac{x+i y}{a+i b}=\frac{x+i y}{a+i b} \times \frac{a-i b}{a-i b} \\ & \frac{z_1}{z_2}=\frac{(a x+y b)+i(a y-x b)}{a^2+b^2} \quad \ldots\ldots \text{(ii)}\end{aligned}$$

From Eq. (i), we get

$$\begin{aligned} & \text{LHS }=\left|z_1+z_2\right|^2=|x+a+i(y+b)|^2 \\ & =(x+a)^2+(y+b)^2 \\ & =x^2+a^2+2 a x+y^2+b^2+2 b y \\ & \text{RHS }=|x+i y|^2+|a+i b|^2 \\ & =x^2+y^2+a^2+b^2 \\ & \because \text{ LHS }=\text{ RHS } \\ & \Rightarrow x^2+y^2+a^2+b^2+2(a x+b y) \\ & =x^2+y^2+a^2+b^2 \\ & \Rightarrow \quad 2(a x+b y)=0 \Rightarrow a x+b y=0\end{aligned}$$

Put $ax+by=0$ in Eq. (ii),

$\frac{z_1}{z_2}=\frac{i(ay-xb)}{a^2+b^2}$, which is purely imaginary.

Question68

A real value of x will satisfy the equation, $\left(\frac{3-4ix}{3+4ix}\right)=\alpha-i\beta,(\alpha,\beta$ are real $)$, if

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Options:

A. $\alpha^2-\beta^2=-1$

B. $\alpha^2-\beta^2=1$

C. $\alpha^2+\beta^2=1$

D. $\alpha^2-\beta^2=2$

Answer: C

Solution:

$$\begin{aligned}
&\text{LHS }=\frac{3-4ix}{3+4ix}\times\frac{3-4ix}{3-4ix}\\
&=\frac{9+16i^2x^2-24ix}{9-(16i^2x^2)}=\frac{9-16x^2-24ix}{9+16x^2}\\
&=\frac{9-16x^2}{9+16x^2}-\frac{24ix}{9+16x^2}=\alpha-i\beta=\text{ RHS}\\
&\therefore\quad\alpha=\frac{9-16x^2}{9+16x^2}\\
&\beta=\frac{24x}{9+16x^2}\\
&\Rightarrow\quad a^2+\beta^2=\frac{\left(9-16x^2\right)^2+(24x)^2}{(9+16x^2)^2}\\
&=\frac{\left(9+16x^2\right)^2}{(9+16x^2)^2}=1\\
&\therefore\quad\alpha^2+\beta^2=1
\end{aligned}$$

Question69

What is the value of $(1-i\sqrt{3})^9$ is equal to

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Options:

A. 2^9

B. -2^9

C. $2^9 i$

D. $-2^9 i$

Answer: B

Solution:

$$\begin{aligned}(1-i\sqrt{3})^9 &= 2^9\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^9 \\ &= 2^9\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)^9 = 2^9\left(e^{-i\pi/3}\right)^9 \\ &= 2^9\left(e^{-i3\pi}\right) = 2^9(\cos 3\pi - i\sin 3\pi) \\ &= 2^9\left((-1)^3-0\right) = -2^9\end{aligned}$$

Question70

$\left(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{6}+\sqrt{2}}{4}i\right)^{2020}$ **is equal to**

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Options:

A. $\frac{1}{2}+\frac{\sqrt{3}}{2}i$

B. $\frac{-1}{2}+\frac{\sqrt{3}}{2}i$

C. $\frac{-1}{2}-\frac{\sqrt{3}}{2}i$

D. $\frac{1}{2}-\frac{\sqrt{3}}{2}i$

Answer: D

Solution:

$$\left(\sqrt{2}\frac{(\sqrt{3}-1)}{2.2}+i\sqrt{2}\frac{(\sqrt{3}+1)}{2.2}\right)^{2020}$$

$$=\left(\frac{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)}{\sqrt{2}}+i\frac{\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)}{\sqrt{2}}\right)^{2020}$$

$$=\left(\frac{\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)}{1}+\frac{i\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)}{1}\right)^{2020}$$

$= \left(\left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right) + i\left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \left(\frac{1}{\sqrt{2}} \cdot \frac{1}{2}\right)\right)\right)^{2020}$

$= (\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ) + i(\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ))^{2020}$

$= (\cos 75^\circ + i \sin 75^\circ)^{2020}$

$= \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12}\right)^{2020}$

$= \left(\cos \frac{5\pi}{12} \times 2020 + i \sin \frac{5\pi}{12} \times 2020\right)$ [De-moivre]

$= \left(\cos \frac{2525\pi}{3} + i \sin \frac{2525\pi}{3}\right)$

$= (\cos(2525 \times 60)^\circ + i \sin(2525 \times 60)^\circ$

$= (\cos(n\pi - 60)^\circ + i \sin(n\pi - 60)^\circ)$, where n = 842

$= ((-1)^n \cos 60^\circ + i(-\sin 60^\circ))$

$= (\cos 60^\circ - i \sin 60^\circ)$ [$\because$ n = even]

$= \frac{1}{2} - i\frac{\sqrt{3}}{2}$

Question71

If $z_1 = 2 + 3i$ and $z_2 = 3 + 2i$, where $i = \sqrt{-1}$, then $\begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix} \begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_2 & z_1 \end{bmatrix}$ is equal to

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Options:

A. $13I$

B. I

C. $26I$

D. Zero matrix

Answer: C

Solution:

$$z_1 = 2 + 3i, z_2 = 3 + 2i$$
$$\Rightarrow \begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix} \begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_2 & z_1 \end{bmatrix}$$
$$= \begin{bmatrix} |z_1|^2 + |z_2|^2 & 0 \\ 0 & |z_1|^2 + |z_2|^2 \end{bmatrix}$$
$$\Rightarrow \quad |z_1|^2 = 4 + 9 = 13$$
$$\Rightarrow \quad |z_2|^2 = 9 + 4 = 13 = \begin{bmatrix} 26 & 0 \\ 0 & 26 \end{bmatrix} = 26I$$

Question72

The radius of the circle represented by $(1+i)(1+3i)(1+7i)=x+iy$ is $(i=\sqrt{-1})$

.

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Options:

A. 1000

B. $10\sqrt{10}$

C. 10000

D. 100

Answer: B

Solution:

$$\begin{aligned} & (1+i)(1+3i)(1+7i) &= x+iy \\ \Rightarrow & (-2+4i)(1+7i) &= x+iy \\ \Rightarrow & -30-10i &= x+iy \\ \Rightarrow & |x+iy| &= \sqrt{30^2+10^2} \\ & = \sqrt{1000} = 10\sqrt{10} & \end{aligned}$$

Question73

If $1, \alpha_1, \alpha_2, \alpha_3$ and α_4 are the roots of $z^5-1=0$ and ω is a cube root of units, then $(\omega-1)(\omega-\alpha_1)(\omega-\alpha_2)(\omega-\alpha_3)(\omega-\alpha_4)+\omega$ is equal to

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Options:

A. 0

B. -1

C. -2

D. 1

Answer: C

Solution:

$\because 1, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of $z^5 - 1 = 0$

$$\Rightarrow \quad (z-1)(z-\alpha_1)(z-\alpha_2)(z-\alpha_3)(x-\alpha_4) = z^5 - 1$$
$$\Rightarrow \quad (\omega-1)(\omega-\alpha_1)(\omega-\alpha_2)(\omega-d_3)(\omega-\alpha_4) = \omega^5 - 1$$
$$(\omega-1)(\omega-\alpha_1)(\omega-\alpha_2)(\omega-\alpha_3)(\omega-\alpha_4)+\omega$$
$$= (\omega^5 - 1) + \omega = \omega^2 - 1 + \omega$$
$$= -1 - 1 = -2$$

Question74

If $a > 0$ and $z = x + iy$, then$\log_{\cos^2\theta}|z-a| > \log_{\cos^2\theta}|z-ai|, (\theta \in R)$implies

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Options:

A. $x > y$

B. $x < y$

C. $x + y = \cos\theta$

D. $x + y < 0$

Answer: A

Solution:

$a > 0, z = x + iy$
$\log_{\cos^2\theta}|z-a| > \log_{\cos^2\theta}|z-ai|$
$0 < \cos^2\theta < 1$
So, $|z-a| < |z-ai|$
$\Rightarrow \quad (x-a)^2 + y^2 < x^2 + (y-a)^2$
$\Rightarrow x^2 + a^2 - 2ax + y^2 < x^2 + y^2 + a^2 - 2ay$
$\Rightarrow -2ax < -2ay$
$\Rightarrow \quad x > y$

Question75

If one root of the equation $ix^2 - 2(i+1)x + (2-i) = 0$ is $(2-i)$, then the other root is

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Options:

A. $-i$

B. $2+i$

C. i

D. $2-i$

Answer: A

Solution:

$ix^2-2(i+1)x+(2-i)=0$ ..... (i)

Let the other roots be α.

$[x-(2-i)][x-\alpha]=0$
$\Rightarrow x^2-(2-i+\alpha)x-(\alpha(2-i))=0$
{comparing with Eq. (i)
$\Rightarrow ix^2-(2i+1+\alpha i)x-[\alpha(2i+1)]=0$
$\Rightarrow \quad 2i+1+\alpha i=2i+2$
$\Rightarrow$
$\alpha i=1$
$\Rightarrow \quad \alpha=\frac{1}{i}=-i$

Question76

If $|z-2|=|z-1|$, where z is a complex number, then locus z is a straight line

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Options:

A. Parallel to X - axis

B. Parallel to Y-axis

C. Parallel to $y=x$

D. Parallel to $y=-x$

Answer: B

Solution:

$\because |z-2|=|z-1|$

Let $z=x+iy$, then

$\Rightarrow \quad |x+iy-2|^2=|x+iy-1|^2$
$\Rightarrow \quad (x-2)^2+y^2=(x-1)^2+y^2$
$\Rightarrow \quad x^2-4x+4=x^2-2x+1$
$\Rightarrow \quad -2x+4x=4-1 \Rightarrow x=3/2$

Which is parallel to Y-axis.

Question77

If $\left(\frac{1+i}{1-i}\right)^m=1$, then m cannot be equal to

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Options:

A. 1934

B. 2024

C. 2172

D. 10^{100}

Answer: A

Solution:

$\left(\frac{1+i}{1-i}\right)^m=1 \Rightarrow\left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^m=1$

$\Rightarrow\left(\frac{1-1+2i}{1+1}\right)=1$

$\Rightarrow i^m=1$

$\Rightarrow m$ is multiple of 4 $\left[\because i^{4n}=1, \forall n \in N\right]$

Option (a) is not a multiple of 4 .

$\therefore \quad m \neq 1934$

Question78

$(\sin \theta-i \cos \theta)^3$ **is equal to**

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Options:

A. $i^3(\cos 3\theta+i \sin 3\theta)$

B. $\cos 3\theta+i \sin 3\theta$

C. $\sin 3\theta-i \cos 3\theta$

D. $(-i)^3(\cos 3\theta+i \sin 3\theta)$

Answer: D

Solution:

$(\sin \theta - i \cos \theta)^3 = (-i)^3 (\cos \theta + i \sin \theta)^3$

$= (-i)^3 (\cos 3\theta + i \sin 3\theta)$ [by De-Moivre theorem]

Question79

Real part of $(\cos 4 + i \sin 4 + 1)^{2020}$ is

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Options:

A. $2^{2020} \cos^{2020} 2 \cos 2020$

B. $2^{2020} \cos^{2020} 2 \cos 4040$

C. $2^{1020} \cos^{2020} 2 \cos 4040$

D. $2^{2020} \cos^{2020} 1 \cos 2020$

Answer: B

Solution:

Real part of $(\cos 4 + i \sin 4 + 1)^{2020} =$ Real part of $\left(2 \cos^2 2 + i 2 \sin 2 \cos 2\right)^{2020}$

$$\begin{aligned} &= \text{Real part of } 2^{2020} \cos^{2020}(2)(\cos 2 + i \sin 2)^{2020} \\ &= \text{Real part of } 2^{2020} \cos^{2020}(2)(\cos 4040 + i \sin 4040) \\ &= 2^{2020} \cos^{2020} 2 \cdot \cos 4040 \quad [z + \bar{z} = 2 \operatorname{Re}(z)] \end{aligned}$$
